FINAL REPORT

HOUSING AS CONSUMPTION AND INVESTMENT

Frank T. Denton, A. Leslie Robb, and Byron G. Spencer

McMaster University

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The work reported herein has been carried out under contract with the Canada Mortgage and Housing Corporation. We thank CMHC for suggesting and supporting the project. We also thank Christine H. Feaver, who was responsible for all of the computer work involved in the project.

ABSTRACT

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A theoretical model is developed in which housing is treated as both a consumption good and an investment good. A multiperiod Stone-Geary type of utility function is first specified, and then reduced to an equivalent two period form. Rental and owned housing are incorporated into the two-period function as separate arguments, along with nonhousing goods. The utility function is specified also to include future assets, as represented by both their expected value and their variance, thus allowing for owned housing to be treated as a risky component of the household investment portfolio. A linear expenditure system is derived from the utility model, and the system is estimated using microdata from the 1982 Statistics Canada Survey of Family Expenditures, together with other data. A tenure-choice function is appended to the model and estimated also, as a probit function. Suggestions are made with regard to future extensions of the research and the possible acquisition of new types of data that would permit the estimation of more comprehensive models. Particular emphasis is placed on the importance of geographic identification, price variables, asset information, and information about households that change residence.

EXECUTIVE SUMMARY

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The main purposes of this study were (1) to develop a theoretical model of how households make their choices between renting and owning, and how they determine their levels of expenditure on housing and nonhousing goods, (2) to appraise the available statistical data from the point of view of their usefulness in estimating such a model, (3) to estimate a feasible version of the model, using econometric techniques and whatever data seemed most appropriate, and (4) to make suggestions as to the possible acquisition of new data in the future that would permit a more refined version of the model to be estimated. A basic requirement of the type of model to be considered was that it should view housing in its dual roles as consumption and investment.

The study begins with a brief review of relevant literature on the theory and econometric modelling of housing consumption and investment. It then proceeds to develop a basic theoretical framework for considering housing decisions by individual households. The concept of a household utility function is introduced, in accordance with standard practice in economic theory. A household is assumed to choose between the consumption of different goods, and between current consumption and future consumption, in such a way as to achieve the highest level of utility available to it, given its assets, income prospects, and other characteristics, and given also the set of prices that it faces. The original form of utility function considered takes account explicitly of each year in the household's planning period. However, it is shown that the multiyear function can be collapsed into a two-period

form, one period representing the current year and the other one representing all future years combined.

The basic theoretical utility or decision-making framework having been developed, housing is then introduced explicitly into this framework. Households are assumed to choose between renting and owning in light of their preferences, their asset-income position, and relative prices. In addition, they view owner-occupied housing as a form of investment, and in this regard they are influenced by the uncertainty about future changes in housing prices. Households are assumed to prefer safe to risky assets, other things equal, and price uncertainty in the housing market thus acts as a partial offset to their preferences for owning. In terms of economic theory, the innovation in this part of the study is the incorporation into a standard utility function of the notion of portfolio choice, housing being viewed as a component of the household's investment portfolio as well as a source of shelter services.

The household utility framework, with housing incorporated, is then used to derive a system of expenditure equations. The system is a variant of the well known "linear expenditure system" used in theoretical and applied economics. It represents the manner in which consumers allocate their wealth between current expenditure and saving, or future expenditure, and how they allocate their current expenditure total between housing and nonhousing goods. Separate sets of expenditure equations are specified for owners and renters. The concept of housing as a risky asset is reflected in the particular form of the equations for owners.

A tenure-choice function is also developed within the basic

consumer utility framework. This function represents the choice between renting and owning as a function of household preferences, assets, income prospects, and demographic composition. Its role in the overall model is to determine the probabilities that a household with given characteristics will own or rent its home. A tenure choice having been made by the household, the expenditure equations then determine how much the household will spend on rented or owned housing.

The data available for estimating an expenditure and tenure-choice system of equations of the kind represented by the theoretical model is considered and the basic choice is the set of microdata provided by the 1982 Statistics Canada Family Expenditures Survey (FAMEX). The FAMEX data set includes information on income, household demographic characteristics, value of owned dwellings, mortgage principal outstanding, and expenditure in various shelter and nonshelter categories. FAMEX is, in fact, the only source of data with broad enough coverage for the purpose at hand. To provide other information required for estimating the model, the FAMEX data are supplemented by the use of price estimates for local housing markets from the Royal LePage Survey of Canadian House Prices, values from the Statistics Canada consumer price index series, and some interest rate data from other sources.

The theoretical model is adapted to obtain a form suitable for econometric estimation with the available FAMEX and other data. The model so adapted assumes a three-stage decision process on the part of households. At the first stage, a household

decides whether to own or rent. At the second stage it chooses its level of total expenditure for the current year, and implicitly its level of saving. At the third stage it allocates total expenditure among shelter and nonshelter categories.

The econometric model is estimated and the results interpreted and discussed. Separate equations are provided for total household expenditure, shelter expenditure, and nonshelter expenditure. In addition, nonshelter expenditure is broken into three categories -- food, clothing, and other nonshelter. Separate expenditure equations are provided also for these three categories. In all cases, equations are estimated separately for renters and owners. The model thus consists of six expenditure equations for renters and six for owners, plus the tenure-choice equation, or thirteen equations in total. Values for parameters of special interest are extracted from the equations and discussed. Particular attention is paid to the differences in expenditure patterns between renters and owners, and among households of different ages (as represented by age of husband). Attention is paid also to the estimated effects of housing price uncertainty on owners' expenditure patterns.

A key concept in the treatment of owners' expenditure is the concept of user cost. Instead of accepting the shelter expenditures reported by FAMEX, shelter expenditures for owner households are recalculated to allow for the opportunity cost of capital embodied in housing, as well as for current operating costs, including taxes, insurance, depreciation, and other components. Total expenditure for owner households is similarly adjusted to reflect shelter expenditures calculated on the user-cost basis.

The tenure-choice equation is estimated by the statistical procedure known as probit estimation. The equation is then used to calculate some illustrative probabilities of owning or renting for households with different sets of characteristics. The influences of wealth, age, and household demographic composition are made clear by these calculations.

The report concludes with a statement of suggestions for further research and possible improvements to the data base for future model estimation. It is noted that additional data from FAMEX could be used and that future extensions to the range of subject matter covered by FAMEX surveys might be considered. Emphasis is given to the importance of having geographic identification so that households can be associated with particular local housing markets, and data from non-FAMEX sources for these areas can be drawn upon to supplement the FAMEX base. Further in this regard, emphasis is placed on the importance of being able to use price data (both for housing and nonhousing goods) in future analyses, this being dependent on geographic identification of individual households, probably at the level of census metropolitan areas. The desirability of coupling more extensive information about household assets with the present set of variables provided by FAMEX is noted too. Finally, it is suggested that consideration might be given to the acquisition of statistical information specifically about <u>mover</u> households -- ones that have changed residence during the course of a year. The latter type of information, if properly specified, would permit the incorporation of transaction costs into future econometric modelling of housing decisions.

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HOUSING AS CONSUMPTION AND INVESTMENT

1. INTRODUCTION

This report represents the final statement of work carried out and results obtained in the project "Housing as Consumption and Investment," which was conceived and supported by the Canada Mortgage and Housing Corporation. The project commenced early in 1986 and continued throughout the year. A progress report was submitted in July. In general terms, the objectives of the project were (1) to formulate an empirically implementable model of housing demand that would recognize that the housing decision is both an investment decision and a consumption decision, (2) to examine available microdata files and suggest what additional data would be necessary for full estimation of such a model, and (3) to do such model estimation as might be feasible with microdata that now exist. We have sought to satisfy each of these objectives within the time available. As is so frequently the case, the project has generated many ideas, not all of which could be pursued by the time that it was necessary to conclude. We hope to be able to follow up some of these ideas in the future. The acquisition of new data would be helpful in further work but we believe that there is more that could be done even with existing data. In any event, we hope that what we have been able to accomplish will be interesting and helpful to persons concerned with the housing consumption-investment issue, and that the framework that we have developed will be suggestive of how future research in this area might proceed.

The structure of the report is as follows. Some of the literature relevant to the project is discussed briefly in section 2,

with a view to placing our work in the context of that literature. A theoretical multiperiod household utility function is proposed in section 3 and it is then shown that the multiperiod function is equivalent to a two-period function, the first period representing the present and the second period representing the entire future of the household. Housing is introduced explicitly into the two-period utility function in section 4. Rental and owned housing are treated as separate and alternative commodities, and the notion of owned housing as a portfolio asset with uncertain price variability is introduced also into the utility function; the theoretical foundation for studying housing as both consumption and investment is thus established. Housing as a risky asset and the user cost of housing are among the cental ideas introduced into the theoretical model. A set of theoretical expenditure equations is derived from the basic utility function in section 5 and a tenure-choice function is derived in section 6. The latter represents the probability that a household will own its home, based implicitly on its comparison of the maximum utility levels available to it under the alternative renting and owning options.

The data used in the subsequent estimation of the theoretical expenditure and tenure-choice functions are described and discussed in section 7. The principal set of data is microdata from the Canadian Family Expenditure Survey carried out by Statistics Canada in February and March of 1983, and relating to the year 1982. These data are supplemented by data from other sources, including especially estimates from the annual Royal LePage Survey of Canadian House Prices.

A version of the theoretical model that can be estimated

using the available data is developed in section 8. Estimates of key parameters or parameter functions are then derived from econometric estimates of the expenditure equations of the model, and these are presented and interpreted in section 9. (The econometric equations themselves are provided in full detail in a series of tables in the Appendix to the report.) Some calculations of ownership probabilities are presented and discussed in section 10, based on the estimated tenure-choice function. (The estimated tenure-choice function is also provided in the Appendix.)

The final section, section 11, sets forth our ideas about the directions that future work might take in the area of housing consumption-investment models and analysis. It also provides a statement of what we think are important issues relating to the present and possible future availability of statistical information for use in studies of this kind. Our aim in this concluding section is to put on record the views that we have formed during the course of the project as to the most promising avenues for future research and to provide some guidance for the future use of existing survey data and possible initiatives to obtain new types of data.

2. THE PROJECT IN RELATION TO EXISTING LITERATURE

We present now a brief review of some of the existing literature that relates to our project, and which we have found helpful. The review is not intended to be anything like exhaustive; the aim is merely to provide a background against which our work can be seen and to indicate in a general way how our work relates to previous work by others.

Most studies relating to housing demand have focussed on either the amount of housing consumed or the decision to own or rent. Much of the attention in these studies has been given especially to the estimation of income and price elasticities of demand. These studies have been reviewed recently by Mayo (1981) and earlier by de Leeuw (1971). We have found only one paper, a purely theoretical one by Henderson and Ioannides (1983), which focuses specifically on the topic that is of special interest here -- namely, the dual characteristics of housing as both a consumption good and an investment good. At the same time, various analytical features and characteristics of housing that have been emphasized by other authors are of interest for our purposes.

Our work starts from a number of basic premises: that housing decisions should be viewed in a multiperiod context, and one should allow for changes in life cycle circumstances (e.g., in terms of income and family size and age composition) that may affect housing choice; that housing expenditures represent a major portion of family expenditures and hence should be viewed in the context of overall expenditures, through the use of an expenditure system; that the equity that a typical family has in housing is both its major investment and a particularly volatile part of

its investment portfolio, and should be recognized as such; and that moving is costly, especially for homeowners, and hence can be expected to occur relatively infrequently. We review briefly selected items from the literature as they relate to each of these premises.

That consumers should make decisions regarding intertemporal consumption allocations that are independent of the time flow of income is, of course, at the very heart of the life cycle hypothesis. Yaari (1964) formalized many of the basic intertemporal ideas, although he was not concerned with housing as such, nor even with durable goods. Somewhat later, and again without specific concern for housing, Diewert (1974) provided a general theoretical analysis of the intertemporal problem which incorporated durable goods. Artle and Varaiya (1978) developed a model explicitly concerned with housing, and derived life cycle tenure and related choices in circumstances in which individuals cannot become homeowners without first saving the required downpayment. However, their model is based on the restrictive assumption that the opportunity cost of owning is less than the cost of renting. Henderson and Ioannides (1983) developed a more general model, also concerned, at least in part, with tenure choice over the life cycle.

A typical family's demographic composition changes markedly over its life cycle and would, of course, be expected to influence its housing consumption. Explicit modelling of demographic variables in demand analysis was addressed by Pollak and Wales (1981) and subsequently by Barnes and Gillingham (1984), who were concerned specifically with shelter as one of their expenditure cate-

gories.

Expenditures on housing are clearly important in family budgets, and hence there is reason to view them in the context of an explicit expenditure system -- as one among a number of alternative expenditures collectively constrained by the available budget. This is the approach pioneered by Stone (1954) and explored subsequently by Pollak and Wales (1969), among others. In subsequent work this general approach has been extended in various ways, including the explicit recognition of housing expenditures; see, for example, Cronin (1982), Barnes and Gillingham (1984) and Andrikopoulos and Brox (1984). (We note, however, that Andrikopoulos and Brox are concerned only with the allocation of housing expenditures among various housing types, and not with the allocation between housing in total and non-housing.)

Various authors have viewed housing tenure decisions in the context of portfolio choice models; Goodwin (1986), Ortmeyer and Peek (1986) and Poloz (1986) could all be classified in this way. Others have been specifically concerned with the potential capital gains associated with house price appreciation, though not always in the context of a model of portfolio behaviour. See, for example, Dougherty and Van Order (1982), Gillingham and Hagemann (1983), Henderson and Ioannides (1983), and Rosen, Rosen, and Holtz-Eakin (1984). Of these studies, the last three have focused specifically on the relationship between tenure choice and the inevitable uncertainty associated with future house prices, but only Gillingham and Hagemann are concerned with estimation within an expenditure system context.

Equity in housing is <u>the</u> major nonhuman asset of typical

families and an asset whose value is particularly volatile. A number of papers have been concerned with the housing demand implications of inflation, risk, and the tax treatment of housing investment, or the interplay among them; see, for example, Schwab (1982), Henderson and Ioannides (1983), Dynarski and Sheffrin (1985), Goodwin (1986), and Mills (1986).

Changing homes, whether from one rental unit to another, from one owned unit to another, or between rental and owned units, is clearly expensive, especially for homeowners. However, there are other barriers to movement, especially into the class of homeowners. Among the potentially important barriers is that of capital market imperfections, implying that low income (and often young) families will simply be unable to become homeowners as soon as they might wish. Analyses relating generally to capital market imperfections have been provided by Dolde and Tobin (1971), and in a housing context by Artle and Varaiya (1978). Others have been concerned with such imperfection in their empirical work; we mention King (1980), Dynarski and Sheffrin (1985), and Henderson and Ioannides (1986). Dynarski and Sheffrin, for example, find evidence that transitory income has an important role to play in the housing purchase decision.

Finally, we turn to an overview of some considerations relating to the estimation of housing demand. Since the choice is inevitably made simultaneously, the model that determines tenure choice should, in principal, also determine housing quantity. While this point has frequently been recognized in model development, at the model estimation stage concern has often been limited

to either the tenure choice decision or the quantity decision. (An estimation problem here is that looking at housing quantity alone may lead to selection bias, whereas looking at tenure choice alone does not.) Among the papers whose empirical estimates are concerned only with tenure choice are Bossons (1978), Boehm (1981), and Rosen, Rosen, and Holtz-Eakin (1984), while the housing quantity decision alone is considered in Goodman and Kawai (1986). (The main concern of Goodman and Kawai is with issues relating to functional form and sample selection.) A number of recent papers have been concerned, in both theoretical and empirical work, with the simultaneous determination of housing tenure and quantity; see, for example, Lee and Trost (1978), Rosen (1979), King (1980), Gillingham and Hagemann (1983), Goodman (1985), Dynarski and Sheffrin (1985), and Henderson and Ioannides (1986).

With the foregoing comments as background, we turn to the development of our own model.

3. THEORETICAL UTILITY FRAMEWORK

It was apparent at the outset of the project that some form of multiperiod optimizing framework was appropriate for the study of housing as both a consumption good and an investment good. However, it was apparent also that some simplifications and restrictions would be required if the framework were to be useful for the development of a model that could actually be estimated with cross-section survey data. We proceed now, in the present section, to define a multiperiod utility function for an individual household and to show how this function can be viewed as equivalent to a two-period ("now and hereafter") function. In the next section, we adapt the two-period function to accommodate housing as a durable good and introduce uncertainty about the price of housing explicitly into the utility function.

A Multiperiod Utility Function

To begin with, then, assume a (strongly separable) multiperiod utility function of the form

$$(3.1) \quad U_0 = \sum_{t=0}^{T} \delta(t) u_t$$

where U_0 is perceived multiperiod utility of a household at time 0, u_t is the value of a single-period utility function for period t, $\delta(t)$ is a time-preference discount function defined such that $\delta(0) = 1$, and T is the terminal point of the planning period. The function u_t is assumed to be of the Stone-Geary form and is defined by

$$(3.2) \quad u_t = \sum_{i=1}^{N} \beta_{it} \ln(x_{it} - \bar{x}_i)$$

where x_{it} is consumption per capita of good i in period t and $\bar{x}_i$ is minimum necessary consumption of good i. The space of the x variables is assumed to be continuous in the relevant range.

We may think of the individual period as being a year. The planning period then consists of T+1 years (t=0,1,2,...,T). In a life cycle model with the date of death known with certainty and no bequest motive, we might think of T as the last year of life. However, this is unduly restrictive for our purposes. We simply regard T as defining the planning horizon. It may be that an individual or a household has a finite and short planning period, a finite and long planning period, or even an infinite planning period (or at least behaves as if the planning period were of infinite length). The planning period might be equal to average life expectancy at time 0 or it might be longer. If it is longer, the justification could be that an individual may live longer than the average and does not want to use up all of his resources in case that happens, or it could be that a longer period implies a bequest motive. A third possibility that could give rise to a planning period longer than a reasonable life expectancy is provided by the concept of consonant dissonance, as found in psychology and adapted for application in economics by Ackerloff and Dickens (1982): an individual may choose to believe that he will live much longer than the average even though objective evidence indicates otherwise. In any event, the choice of T by an individual decision-making unit is an open issue.

To firm things up a bit, we may think of the decision-

making unit as being a household consisting of a husband and wife, together with some specified number of children who are born, mature, and leave home at various points in the husband-wife life cycle. The husband and wife make a joint optimizing decision at time 0 and then reoptimize subsequently at times 1,2, etc. In practice, wives are about two years younger than husbands, on average, but we ignore this difference: we assume that both are of the same age.

It is convenient for the argument that follows to normalize the β parameters of equation (3.2). Without any loss of generality, we may write

$$(3.3) \quad \sum_{i=1}^{N} \beta_{it} = 1 \qquad (t=0,1,\ldots,T)$$

and note that this implies

$$(3.4) \quad \sum_{t=0}^{T} \sum_{i=0}^{N} \delta(t)\beta_{it} = \sum_{t=0}^{T} \delta(t)$$

The household is assumed to have wealth (human wealth plus tangible assets) of present value W_0 and its optimizing problem is then to choose a consumption set for each period so as to maximize U_0 subject to the wealth constraint

$$(3.5) \quad W_0 = \sum_{t=0}^{T} \sum_{i=1}^{N} (1+\rho)^{-t} n_t x_{it} p_{it}$$

where ρ is an (after-tax) rate of interest, p_{it} is the price of good i in year t, and n_t is the number of consuming units in the

household in year t. (The solution is also constrained by $x_{it} > \bar{x}_i$, i=1,2,...,N, t=0,1,...,T, but these constraints will be satisfied automatically, given the form of the utility function and assuming a feasible solution exists.) We assume for purposes of the argument that ρ and all p_{it} are known with certainty (or that the household has expectations about them which it treats as if they were certain). It is convenient to think of n_t as being the number of adult-equivalent consuming units, with children of different ages given appropriate weights. The problem is of the standard equality-constrained maximizing form: differentiating U_0 with respect to x_{it} (i=1,2,...,N; t=0,1,...,T) subject to equation (3.5), setting the first derivatives equal to 0, and making use of (3.3) and (3.4), the solution may be written in the form

$$(3.6) \qquad x_{it} = \bar{x}_i + \pi_{it}(W_0 - \bar{W}_0)/n_t p_{it} \qquad (i=1,2,\ldots,N; t=0,1,\ldots,T)$$

where $\bar{W}_0$ is minimum necessary wealth (the wealth that must be set aside to meet minimum necessary consumption levels $\bar{x}_i$ in the present and all future years). $\bar{W}_0$ is given by

$$(3.7) \qquad \bar{W}_0 = \sum_{t=0}^{T} \sum_{i=1}^{N} (1+\rho)^{-t} n_t \bar{x}_i p_{it}$$

and (using the normalization restrictions of (3.3)) π_{it} is given by

$$(3.8) \qquad \pi_{it} = (1+\rho)^t \delta(t) \beta_{it} / \sum_{t=0}^{T} \delta(t) \qquad (i=1,2,\ldots,N; t=0,1,\ldots,T)$$

Multiplying both sides of equation (3.6) by $n_t p_{it}$, we obtain

$$(3.9) \qquad e_{it} = \bar{e}_{it} + \pi_{it}(W_0 - \bar{W}_0) \qquad (i=1,2,\ldots,N; t=0,1,\ldots,T)$$

where the expenditure variables e_{it} and $\bar{e}_{it}$ are defined by

$$(3.10) \qquad e_{it} = n_t x_{it} p_{it}$$

$$(3.11) \qquad \bar{e}_{it} = n_t \bar{x}_i p_{it}$$

Equation (3.9) will be recognized as a form of the well known linear expenditure system. Our particular interest is in the expenditure-wealth equations for year 0:

$$(3.12) \qquad e_{i0} = \bar{e}_{i0} + \pi_{i0}(W_0 - \bar{W}_0) \qquad (i=1,2,\ldots,n)$$

Note that $\pi_{i0} = \beta_{i0} / \sum_{t=0}^{T} \delta(t)$. Note also that if the discount function is specialized to $\delta(t) = (1+\delta)^{-t}$, then π_{i0} is equal to $\beta_{i0}/(T+1)$ for $\delta=0$ (no discounting); if $\delta>0$ and $T\to\infty$, then $\pi_{i0}\to\beta_{i0}\delta/(1+\delta)$.

An Equivalent Two-Period Utility Function

We now show that the multiperiod utility function defined by equations (3.1) and (3.2) is equivalent to a particular form of two-period function. Let us first redefine the U_0 function as

$$(3.13) \qquad U_0 = \sum_{i=1}^{N} \beta_{i0} \ln(x_{i0} - \bar{x}_i) + \gamma \ln(a_1 - \bar{a}_1)$$

where a_1 is defined implicitly by

$$(3.14) \qquad \sum_{t=1}^{T} (1+\rho)^{-t} n_t a_1 = W_0 - \sum_{i=1}^{N} n_0 x_{i0} p_{i0}$$

The interpretation of a_1 is as follows: It is a per capita annuity to be received by the household (for each adult equivalent consuming unit) over the whole of the planning period, commencing with

year 1, and its value is determined by the amount of unconsumed wealth remaining after deduction of period-0 consumption expenditure. The left side of equation (3.14) is the present value of the annuity stream; the right side is the present value of household wealth, net of period-0 expenditures.

The definition of $\bar{a}_1$ is similar. It is the corresponding per capita annuity that would have to be received (commencing in year 1) in order for the household to meet future minimum necessary consumption requirements, and is given implicitly by

$$(3.15) \qquad \sum_{t=1}^{T} (1+\rho)^{-t} n_t \bar{a}_1 = \bar{W}_0 - \sum_{i=1}^{N} n_0 \bar{x}_i p_{i0}$$

The explicit definitions obtained from (3.14) and (3.15) are

$$(3.16) \qquad a_1 = (1/k_0)\left(W_0 - \sum_{i=1}^{N} n_0 x_{i0} p_{i0}\right)$$

$$(3.17) \qquad \bar{a}_1 = (1/k_0)\left(\bar{W}_0 - \sum_{i=1}^{N} n_0 \bar{x}_i p_{i0}\right)$$

where $k_0 = \sum_{t=1}^{T} (1+\rho)^{-t} n_t$

The optimizing problem is now that of choosing a current consumption set and a future annuity value so as to maximize (3.13) subject to the constraint

$$(3.18) \qquad W_0 = \sum_{i=1}^{N} n_0 x_{i0} p_{i0} + a_1 k_0$$

(If a feasible solution exists, the constraints $x_{it} > \bar{x}_i$, $i=1,2,\ldots,N$, and $a_1 > \bar{a}_1$ will be satisfied automatically, given the form of the utility function, and can therefore be ignored.) As

before, we impose the convenient normalization $\sum_{i=1}^{N} \beta_{i0} = 1$. The solution can be put into the form

$$(3.19) \qquad e_{i0} = \bar{e}_{i0} + \left(\beta_{i0}/(1+\gamma)\right)\left(W_0 - \bar{W}_0\right) \qquad (i=1,2,\ldots,N)$$

If γ is interpreted as the sum of the time-preference discount factors from year 1 to year T, and recalling that $\delta(0)=1$, we can write

$$(3.20) \qquad 1 + \gamma = 1 + \sum_{t=1}^{T} \delta(t) = \sum_{t=0}^{T} \delta(t)$$

Hence $\beta_{i0}/(1+\gamma) = \pi_{i0}$ and equation (3.19) is identical to equation (3.12). In other words, the linear expenditure system derived from the two-period utility function of equation (3.13) is identical to the system derived from the multiperiod utility function of equations (3.1) and (3.2). It may be noted that this is an application of two-stage budgeting and is possible because the assumed utility function is strongly separable and quasi-homothetic. (See Deaton and Muellbauer, 1980.)

The foregoing is a very convenient result for our purposes. It allows us to use the two-period utility function as a starting point for the modelling of housing consumption-investment decisions in the knowledge that the model to be developed has a foundation that is consistent with that of multiperiod optimization.

4. HOUSING IN THE UTILITY FRAMEWORK

We now adapt the utility function of equation (3.13) to incorporate also the notion of uncertainty in respect of housing as an asset in the household portfolio. The modified utility function has the form

$$
\begin{aligned}
(4.1) \qquad U_0 = {} & \alpha_1\eta \ln(h_{10}-\bar{h}) + \alpha_2(1-\eta)\ln(h_{20}-\bar{h}) \\
& - \alpha_3\epsilon_0 + \sum_{i=1}^{N} \beta_i \ln(x_{i0}-\bar{x}_i) \\
& + \gamma \ln\left(E(\hat{a}_1-\bar{a}_1) - \lambda(\mathrm{Var}(\hat{a}_1-\bar{a}_1))^{1/2}\right)
\end{aligned}
$$

The interpretation of this function is as follows. We assume, as before, that a household consists of a husband and wife plus some specified number of children who are born, mature, and leave home at various points in the husband-wife life cycle. The husband and wife jointly plan household consumption and saving by optimizing or reoptimizing at each year of age. We take the present age to be 0 and assume again that the husband and wife are of the same age.

The choice set for the household now includes x_{i0} $(i=1,2,\ldots,N)$, the set of all nonhousing goods to be consumed (per adult equivalent) in year of age 0 (food, clothing, etc.), h_{10} and h_{20}, representing the consumption of owned and rented housing, respectively, in year of age 0, and $\hat{a}_1$, a per capita annuity variable of the kind defined previously, representing the consumption of all goods at all future ages. The utility function is thus a two-period function, the first period (year 0) being "now" and the second period (year 1) being "hereafter," as in the previous section.

The utility function, being a modified form of the Stone-Geary function, involves "minimum required" consumption levels. The minimum required levels of nonhousing goods are represented by $\bar{x}_i$ $(i=1,2,\ldots,N)$. The minimum required level of housing consumption is $\bar{h}$, and is assumed to be the same whether the housing is owned or rented. The minimum requirement for future consumption is represented by $\bar{a}_1$. It may be noted that whereas $\bar{x}_i$ and $\bar{h}$ are expressed per adult equivalent consuming unit, and are therefore independent of the current adult-child composition of the household, $\bar{a}_1$ depends on the composition of the household at all future ages (as perceived at age 0 by the husband and wife). Note also that while we have referred to $\bar{x}_i$, $\bar{h}$, and $\bar{a}_1$ as "minimum requirements," it is not necessary to interpret them as minima in a physiological sense; they may be interpreted simply as the coordinates of a shifted origin point in the consumption goods space.

Two other assumptions should be mentioned at this point. First, we abstract from mortality, and from household dissolution generally; the husband and wife make their consumption-saving plans on the assumption that the household will survive to the end of the planning period. Secondly, we adopt the fiction that all expenditures and receipts of income occur at the middle of the year; stock variables are correspondingly dated at the middle of the year. (The middle-of-the-year fiction does no harm to the analysis and is convenient for the interpretation of what, in the real world, are essentially continuous processes in time as if they were discrete-time processes.)

A characteristic of housing which is not shared by most other commodities is that owned and rented housing are all-or-

nothing substitutes for each other. Households do not partly rent and partly own, except in circumstances that are transitory or so unusual that we would wish to ignore them. Accordingly, the utility function incorporates a binary parameter η which turns on or off the particular subfunctions relating to owned and rented housing: η is either 1 (if housing is owned) or 0 (if housing is rented). In other words, with regard to the two types of housing, only corner solutions in the space of consumption goods are permitted. This means that an overall optimizing solution cannot be found by the usual procedure of differentiating the utility function subject to equality constraints and setting the derivatives to zero. In effect, the household has a nonlinear programming problem to solve at each age. Maximum utilities must be calculated for alternative owner-renter states, and the optimum state selected by choosing the greatest of the two utility maxima.

The variable ϵ_0 in the utility function is a binary variable representing the occurrence or nonoccurrence of a move. If the household moves from one dwelling to another in year of age 0, then $\epsilon_0 = 1$; otherwise, $\epsilon_0 = 0$. The effect of the negative term involving ϵ_0 in the utility function is to allow for the assumed disutility of a move, as such (i.e., as distinct from the utility effects of the change in level of housing services with which the move may be associated). Such disutility may stem from the effort by members of the household required to make the move, from personal inconvenience and the disruption of normal household activities, and so on.

The household is subject to the usual present-value wealth

constraint. Total wealth at age 0 is again represented by W_0, and is assumed to consist of nonhuman assets A_0 and human wealth that has present value Y_0:

$$(4.2) \quad W_0 = A_0 + Y_0$$

Nonhuman assets can be of two types, housing and nonhousing, and total nonhuman assets can be positive or negative. If P_0 is the asset price per unit of owned housing at age 0 and H_0 is the quantity of housing owned (equal to zero for a renter household), then

$$(4.3) \quad A_0 = H_0P_0 + B_0$$

where B_0 stands for net nonhuman, nonhousing assets. (For mnemonic convenience, one can think of B as standing for "bonds" if it is positive and for "borrowing" if it is negative.)

Human wealth is defined as the present value of all future earnings and transfer payments, after tax. At interest rate ρ, this present value is given by

$$(4.4) \quad Y_0 = \sum_{t=0}^{T} (1+\rho)^{-t} y_t$$

where y_t is after-tax household earnings and (net) transfers in year t, and T is again the household's planning horizon. For purposes of the model, we assume that y_t is known to the household for t=0,1,...,T.

Now consider the level of total present-value wealth at age 1, as perceived by the household at age 0. The asset price of owned housing a year hence is uncertain; it is regarded by the household as a random variable since it cannot be predicted with

perfect accuracy. The prices of other assets may also be uncertain, of course, but for convenience we take them as given to the household. (Aside from simplification, the justification for this assumption is that the value of housing is likely to be the most volatile component of savings for most households.) Total wealth at age 1, as perceived at age 0, may then be written as

$$(4.5)\quad \hat{W}_1 = (1+\rho)\Big(W_0 - n_0 h_{10}\hat{q}_0 - n_0 h_{20} r_0 - \sum_{i=1}^{N} n_0 x_{i0} p_{i0} - (c_0 + c_0^{*} H_0 P_0 + c_0^{**} H_{-1} P_0)\,\epsilon_0\Big)$$

Anticipated wealth at age 1, $\hat{W}_1$, is written with the symbol ^ to indicate that it is a random variable. The price of owned housing consumption is $\hat{q}_0$, a random variable to be defined below; r_0 is rent (the price of rented housing consumption).

The final expression in the brackets on the right side of equation (4.5) represents the cost of moving; it is operative only if $\epsilon_0 = 1$ (i.e., a move takes place). The moving cost has three components: c_0, representing such costs as the costs of transport of household goods; c_0^{*}, representing such costs as legal fees associated with (and assumed proportional to) the value of the house into which the household moves (if it moves into an owned dwelling; otherwise H_0 will be zero and c_0^{*} inoperative); and c_0^{**}, representing such costs as legal fees and real estate agency fees associated with (and again proportional to) the value of the house out of which the household moves (if it moves out of an owned dwelling; otherwise H_{-1}, representing housing owned in the previous year of age, will be zero). Note that for this purpose a house owned at age −1 is valued at P_0, which is the <u>current</u> asset sale price at

age 0. In practice, the c_0 component may vary with size of household, volume of household durables owned, and other household characteristics, but we ignore this complication and assume it to be constant.

The price of owned housing consumption, $\hat{q}_0$, is defined as the operating cost per housing unit, including depreciation, plus the capital loss associated with holding wealth in the form of housing rather than as a revenue-producing asset (i.e., holding it as H_0P_0 instead of B_0). The present value of the cost of living in an owned dwelling of size H_0 for the current year is $d_0H_0 + P_0H_0 - (1+\rho)^{-1}\hat{P}_1H_0$ where d_0 is the operating cost per unit (including depreciation, P_0H_0 is the initial market value of the dwelling, and $\hat{P}_1H_0$ is the value of the dwelling one year later. Since $\hat{P}_1$ is unknown at age 0, it is treated by the household as a random variable. Now assume that housing services and housing stock are related by a factor of proportionality θ, so that

$$H_0 = \theta h_{10} \tag{4.6}$$

Defining $\hat{q}_0$ by

$$\hat{q}_0 = \left(d_0 + P_0 - (1+\rho)^{-1}\hat{P}_1\right)\theta \tag{4.7}$$

the present-value cost of living in the dwelling may now be written as

$$d_0H_0 + P_0H_0 - (1+\rho)^{-1}\hat{P}_1H_0 = \hat{q}_0h_{10} \tag{4.8}$$

The randomness of $\hat{q}_0$ derives entirely from the randomness of (uncertainty about) $\hat{P}_1$, it will be noted.

As before, we think of the household as if it were to

purchase an annuity, a_1. The annuity must be consistent with the household's wealth at time 1, and is defined implicitly by

$$(4.9) \quad \hat{W}_1 = \sum_{t=1}^{T} (1+\rho)^{1-t} n_t \hat{a}_1$$

or equivalently by

$$(4.10) \quad (1+\rho)^{-1}\hat{W}_1 = W_0 - n_0 h_{10}\hat{q}_0 - n_0 h_{20} r_0 - \sum_{i=1}^{N} n_0 x_{i0} p_{i0} - K_0$$

$$= \sum_{t=1}^{T} (1+\rho)^{-t} n_t \hat{a}_1$$

where $K_0 = (c_0 + c_0^{*} H_0 P_0 + c_0^{**} H_{-1} P_0)\epsilon_0$. In explicit form, $\hat{a}_1$ is defined by

$$(4.11) \quad \hat{a}_1 = (1/k_0)\left(W_0 - n_0 h_{10}\hat{q}_0 - n_0 h_{20} r_0 - \sum_{i=1}^{N} n_0 x_{i0} p_{i0} - K_0\right)$$

where $k_0 = \sum_{t=1}^{T} (1+\rho)^{-t} n_t$

The annuity value is stochastic, in direct consequence of the stochastic nature of $\hat{q}_0$, and more fundamentally as a result of the uncertainty about next year's price of owned housing, $\hat{P}_1$.

We assume that all prices in the system are defined relative to some general price index, the future values of which are known with certainty. (Since the owned housing price is uncertain, we may regard this index as involving either the rental index to represent its shelter component, or else the expected price of owned housing; alternatively, the index may be based only on non-shelter prices. The aim is merely to be able to abstract from "general inflationary" price changes.) The after-tax rate of interest and all income variables in the system are similarly defined in "real" terms. The "real" price of owned housing is determined

as the outcome of a stochastic process and we assume that the household regards this process as a random walk. Thus the consumer thinks of $\hat{P}_1$ as being determined by

(4.12) $\hat{P}_1 = P_0 + \hat{v}_1$

where $\hat{v}_1$ is a random disturbance that is uncorrelated both with its own previous values and with P_0 and all previous values of P, and is normally distributed with mean and variance given by

(4.13) $E(v_1) = 0; \qquad Var(v_1) = \sigma^2$

Let $E(\hat{P}_1|P_0)$ and $Var(\hat{P}_1|P_0)$, or more simply, just $E(\hat{P}_1)$ and $Var(\hat{P}_1)$, be the expected value and variance of next year's owned housing price, conditional on knowledge of P_0. Based on (4.13), these are then given by

(4.14) $E(\hat{P}_1) = P_0; \qquad Var(\hat{P}_1) = \sigma^2$

The owned housing price was defined in equation (4.7), and its expected value may now be given the symbol q_0^* and written as

(4.15) $q_0^* \equiv E(\hat{q}_0) = \left(d_0+P_0-(1+\rho)^{-1}E(\hat{P}_1)\right)\theta = \left(d_0+(\rho/(1+\rho))P_0\right)\theta$

The choice of a unit of measurement for owned housing as an asset (H) is arbitrary and it is convenient to choose one such that $P_0 = 1$. The choice of unit for measuring housing services (h) is also arbitrary and we may make the choice so that $\theta = 1$. (One unit of housing services is then defined as the services resulting from living in one unit of housing asset for one year.) These convenient simplifications allow us to redefine q_0^* as a "user cost:"

(4.16) $q_0^* = d_0 + \rho/(1+\rho)$

We return now to the utility function of equation (4.1), and in particular to the final expression in the function. The expression involves $\hat{a}_1$ and $\bar{a}_1$ The definition of $\hat{a}_1$ is given by equation (4.10) or (4.11) and $\bar{a}_1$ is defined in a similar fashion:

(4.17) $(1+\rho)^{-1}\bar{W}_1 = \sum_{t=1}^{T}(1+\rho)^{-t}n_t\bar{a}_1$ or $\bar{a}_1 = (1/k_0)\bar{W}_1$

where k_0 is as defined previously and $\bar{W}_1$ is the amount of wealth required at time 1 to provide for all future minimum consumption requirements. Taking $\bar{a}_1$ as nonstochastic (implying that future minimum requirements would be satisfied by rental housing, for which the price is assumed known with certainty), the final expression in the utility function may be written as

(4.18) $E(\hat{a}_1-\bar{a}) - \lambda(\mathrm{Var}(\hat{a}_1-\bar{a}_1))^{1/2} = E(\hat{a}_1) - \lambda S(\hat{a}_1) - \bar{a}_1$

where the operator S stands for standard deviation $\left(S(\hat{a}_1) = (\mathrm{Var}(\hat{a}_1))^{1/2}\right)$. The expression can be viewed as a particular form of mean-variance representation of the annuitized wealth portfolio at time 1, as anticipated by the husband-wife decision makers when they choose an optimal consumption-saving program at time 0. Alternatively, the expression can be interpreted in terms of the probability that the actual or realized value of $\hat{a}_1$ (as distinguished from the anticipated value) will exceed the minimum required value $\bar{a}_1$.

Assume that $\hat{a}_1$ is normally distributed (at least in the minds of the husband-wife decision makers): $\hat{a}_1 \sim N\left(E(\hat{a}_1), \mathrm{Var}(\hat{a}_1)\right)$. We may then write

(4.19) $\Pr\left(\hat{a}_1 > \bar{a}_1 \,\middle|\, E(\hat{a}_1) - \lambda S(\hat{a}_1) > \bar{a}_1\right) > F(\lambda)$

where $F(\cdot)$ stands for the standard normal cumulative distribution function. The parameter λ thus represents the "safety level" of the annuitized form of anticipated wealth. The decision makers hold an expectation $E(\hat{a}_1)$ but to protect themselves against retaining too little of their wealth for the future they allow a safety margin equal to λ standard deviations. We assume $\lambda \geq 0$: a large positive value of λ would imply a strong preference for risk aversion; a small positive value would imply a weak preference.

Applying the mean and variance operators to equation (4.11), using (4.7) and (4.14) - (4.16), recalling that $\theta = 1$ and $P_0 = 1$, and rearranging terms, we obtain the following two constraints on the household's optimizing decision:

(4.20) $$W_0 = n_0 h_{10} q_0^* + n_0 h_{20} r_0 + \sum_{i=1}^{N} n_0 x_{i0} p_{i0} + k_0 E(\hat{a}_1) + K_0$$

(4.21) $$\mathrm{Var}(\hat{a}_1) = (n_0 h_{10}/k_0)^2 \sigma^2/(1+\rho)^2$$

Equation (4.21) can be written alternatively as

(4.22) $$S(\hat{a}_1) = (n_0 h_{10} k_0)\sigma/(1+\rho)$$

The household's problem may now be viewed as that of maximizing the utility function of equation (4.1) subject to the restrictions (4.20) and either (4.21) or (4.22). The household must decide whether to rent or own its housing, depending on which alternative yields the greater maximum utility. Under the owner option, it chooses a set x_{i0} (i=1,2,...,N) and h_{10} (and implicitly H_0 and the

mean and variance of $\hat{a}_1$); under the renter option it chooses x_{i0} (i=1,2,...,N) and h_{20}. We are modelling housing as a risky asset in the household portfolio and assuming (for simplicity) that the competing asset (B) is risk-free. Note therefore that $Var(\hat{a}_1) = S(\hat{a}_1) = 0$ if the household chooses to rent; the renter household is constrained only by equation (4.20).

The annuity choice variable, $\hat{a}_1$, and the corresponding minimum requirements variable, $\bar{a}_1$, are functions of anticipated future demographic characteristics of the household (as implied by the definition of k_0). These characteristics are assumed to be determined by the husband and wife independently of (and prior to) the consumption-saving choices. A more ambitious model would allow the planned number and spacing of children to be in the choice set. It would also allow leisure to be viewed as a choice variable, and work-leisure decisions therefore to be endogenous. However, these extensions of the model would complicate it greatly and we do not attempt them. Current demographic characteristics are reflected in n_0, which appears in the wealth and variance constraints.

The maximization of the utility function is subject to the equality constraints represented by equations (4.20) and (4.21) or (4.22). It is also subject to the constraints that only one of h_{10} and h_{20} can have a nonzero value and one of them must have a nonzero value. These latter constraints could be represented formally as

$$(4.23) \quad h_{10}\, h_{20} = 0; \quad h_{10} + h_{20} > 0$$

However, they are caught up in the specification of the utility function and the definition of η, which is unity for an owner

household and zero for a renter household. If $\eta = 1$, the form of the utility function ensures $h_{10} > \bar{h} > 0$ and $h_{20} = 0$ (assuming a feasible solution exists); if $\eta = 0$, the function ensures $h_{20} > \bar{h} > 0$ and $h_{10} = 0$. The constraints $x_{i0} > \bar{x}_i$ (i=1,2,...,N) are also satisfied automatically.

An additional assumption that we have considered is that rental accommodation is not available above a certain level of housing service. (In fact, high-value rental accommodation is apt to be <u>scarce</u> but not nonexistent; however, for theoretical modelling purposes the more restrictive assumption is easier to work with.) Letting h_2^* be the upper bound on rental accommodation, we may write

$$h_{20} \leq h_2^* \tag{4.24}$$

The effect of this restriction would be to force households with high levels of housing expenditure into the owned-dwelling market.

In practice, it has not been feasible to take account of a possible constraint on the availability of high-value rental accommodation in the econometric model that we develop. Moreover, the data set with which we have been working does not allow household moves to be taken account of. In practice, therefore, we are forced to ignore the term involving ϵ_0 in the utility function, and the term K_0 (which also involves ϵ_0) in the wealth constraint. For purposes of deriving demand equations in the next section, we rewrite the utility function of equation (4.1) in the slightly simpler form

$$(4.25)\quad U_0 = \alpha_1\eta\ln(h_{10}-\bar{h}) + \alpha_2(1-\eta)\ln(h_{20}-\bar{h}) + \sum_{i=1}^{N}\beta_i\ln(x_{i0}-\bar{x}_i) + \gamma\ln\left(E(\hat{a}_1)-\lambda S(\hat{a}_1)-\bar{a}_1\right)$$

and the constraints as

$$(4.26)\quad W_0 = n_0h_{10}q_0^* + n_0h_{20}r_0 + \sum_{i=1}^{N}n_0x_{i0}p_{i0} + k_0E(\hat{a}_1)$$

$$(4.27)\quad S(\hat{a}_1) = (n_0h_{10}/k_0)\sigma/(1+\rho)$$

There is a presumption that households prefer owning to renting, other things equal, or at least that they will not prefer renting; hence we expect $\alpha_1 \geq \alpha_2$. There is also a presumption that they will be risk averse, or at least risk neutral; hence $\lambda \geq 0$. Note that if a household chooses to rent (η=0), its portfolio contains none of the risky housing asset, and $S(\hat{a}_1)$ is therefore zero; in other words the constraint (4.27) applies only to owner households.

The utility function of equation (4.25) implies a population of households with identical tastes, regardless of whether the households are owners or renters. An alternative possibility is that owners and renters have different tastes, and should therefore be assigned different utility functions; the fact of ownership or nonownership may itself affect household preferences, and we therefore write, as an alternative to (4.25), the <u>conditional</u> utility functions

$$(4.28)\quad (U_0|\eta=1) = \alpha_1\ln(h_{10}-\bar{h}) + \sum_{i=1}^{N}\beta_{1i}\ln(x_{i0}-\bar{x}_i) + \gamma_1\ln\left(E(\hat{a}_1)-\lambda S(\hat{a}_1)-\bar{a}_1\right)$$

$$(4.29) \quad (U_0|\eta=0) = \alpha_2 \ln(h_{20}-\bar{h}) + \sum_{i=1}^{N} \beta_{2i} \ln(x_{i0}-\bar{x}_i) + \gamma_2 \ln(a_1-\bar{a}_1)$$

The parameters of these two functions are allowed to differ (aside from the minimum necessary consumption levels). In addition, since renters do not hold the risky housing asset, they know with certainty, at time 0, the value of their assets at time 1, and hence the annuity value. We therefore write a_1 for renters without the symbol ^, i.e., as a nonrandom variable. An owner household maximizes $(U_0|\eta=1)$ subject to (4.26) and (4.27). A renter household maximizes $(U_0|\eta=0)$ subject only to (4.26).

To sum up, then, our utility model is a two-period model, but can be viewed as derived within a multiperiod framework. Owned and rented housing are treated separately, and the services of owned housing are distinguished from the housing asset. An owner household is assumed to regard its housing asset as a risky component of its portfolio, and to be risk-averse, or risk-neutral at the least. An owner household maximizes utility subject to a wealth constraint and an uncertainty (variance) constraint. A renter household does not hold the risky housing asset, and therefore maximizes utility subject only to a wealth constraint. The possibility that owner and renter households have different preferences is left open by specifying a utility function in which they are the same and, alternatively, a pair of conditional utility functions in which they are allowed to differ.

5. A DERIVED EXPENDITURE SYSTEM

The combined utility function of equation (4.25) or the separate functions of equations (4.28) and (4.29) give rise to a linear expenditure system, the equations of which can be put into various alternative forms. We proceed now to derive the basic system. The utility functions of (4.28) and (4.29) are less restrictive than that of (4.25) and we use them for the derivation.

The conditional utility function for renters, defined by (4.29), is maximized subject to the wealth restriction represented by (4.26), with $h_{10} = 0$ and $\hat{a}_1$ replaced by a_1, since the future annuity is nonstochastic in the renter case. The present value of minimum necessary wealth at time 0 is defined by

$$(5.1) \qquad \bar{W}_0 = n_0\bar{h}r_0 + \sum_{i=1}^{N} n_0\bar{x}_i p_{i0} + k_0\bar{a}_1$$

Differentiating (4.29) with respect to h_{20}, x_{i0} $(i=1,2,\ldots,N)$, and a_1, under the restriction (4.26), setting the first derivatives to zero, and using (5.1), the resulting system can be expressed in the following form:

$$(5.2) \qquad e_{i0} = \bar{e}_{i0} + \left(\beta_{2i}/(\alpha_2+\beta_2+\gamma_2)\right)(W_0-\bar{W}_0) \qquad (i=1,2,\ldots,N)$$

$$(5.3) \qquad e_{h0} = \bar{e}_{h0} + \left(\alpha_2/(\alpha_2+\beta_2+\gamma_2)\right)(W_0-\bar{W}_0)$$

where $\beta_2 = \sum_{i=1}^{N} \beta_{2i}$ and the e and $\bar{e}$ variables are defined by

$$(5.4) \qquad e_{i0} = n_0 x_{i0} p_{i0}; \qquad \bar{e}_{i0} = n_0\bar{x}_i p_{i0}$$

$$(5.5) \qquad e_{h0} = n_0 h_{10} r_0; \qquad \bar{e}_{h0} = n_0\bar{h}r_0$$

Equations (5.2) and (5.3) express current expenditures as

functions of present-value wealth variables. An alternative form of the system can be obtained by using the identities

$$(5.6) \qquad e_0 = e_{h0} + \sum_{i=1}^{N} e_{i0}; \quad \bar{e}_0 = \bar{e}_{h0} + \sum_{i=1}^{N} \bar{e}_{i0}$$

where e_0 is total expenditure and $\bar{e}_0$ is total minimum necessary expenditure in the current period. Adding the expenditure equations (5.2) and (5.3) and using the definitions in (5.4) and (5.5), we can eliminate $(W_0-\bar{W}_0)$ to obtain

$$(5.7) \qquad e_{i0} = \bar{e}_{i0} + \left(\beta_{2i}/(\alpha_2+\beta_2)\right)(e_0-\bar{e}_0) \qquad (i=1,2,\ldots,N)$$

$$(5.8) \qquad e_{h0} = \bar{e}_{h0} + \left(\alpha_2/(\alpha_2+\beta_2)\right)(e_0-\bar{e}_0)$$

Equations (5.7) and (5.8) represent the well known current-period linear expenditure system. The coefficients in front of the $(e_0-\bar{e}_0)$ terms represent the proportionate shares of discretionary current expenditure going to shelter and to particular nonshelter categories of goods. The parameter γ_2 is not present in this system and the system therefore cannot be used to calculate the share of discretionary-value wealth that is allocated to current-period expenditure. For the latter purpose, one or more relationships extracted from the wealth-based system (5.2) and (5.3) are required.

The system (5.7) and (5.8) consists of N + 1 equations. However, one of these is redundant. Given (5.6), equations (5.7) and (5.8) must sum to e_0 on both the left and right sides. The share of expenditure going to one of the N + 1 goods can therefore be determined residually.

An additional equation can be derived from the wealth-

based system. Summing over equations (5.2) and (5.3) yields

$$(5.9) \qquad e_0 = \bar{e}_0 + \left((\alpha_2+\beta_2)/(\alpha_2+\beta_2+\gamma_2)\right)\left(W_0-\bar{W}_0\right)$$

Total current-period expenditure is thus calculated as minimum necessary total expenditure plus a fraction $(\alpha_2+\beta_2)/(\alpha_2+\beta_2+\gamma_2)$ of present-value discretionary wealth.

Turning now to owners, the conditional utility function of equation (4.28) is maximized subject to the constraints (4.26) and (4.27), with $h_{20} = 0$. Minimum required present-value wealth for owners is symbolized by $\bar{W}_0^*$ and defined by

$$(5.10) \qquad \bar{W}_0^* = n_0\bar{h}q_0^* + \sum_{i=1}^{N} n_0\bar{x}_i p_{i0} + k_0\bar{a}_1$$

The difference between $\bar{W}_0^*$ and $\bar{W}_0$ (as defined in (5.1)) is that in the former, current-period minimum required housing services are priced at the expected owned-dwelling price, whereas in the latter they are priced at the rental price. Otherwise the two minimum-wealth variables are the same. (It is assumed that minimum housing requirements would be satisfied by rental housing in all periods subsequent to period 0, so that these components are the same in $\bar{W}_0^*$ as in $\bar{W}_0$.) Setting first-derivatives to zero and solving, we first obtain

$$(5.11) \qquad e_{i0} = \bar{e}_{i0} + \left(\beta_{1i}/(\alpha_1+\beta_1+\gamma_1)\right)\left(W_0-\bar{W}_0^*-(\lambda\sigma/(1+\rho))n_0\bar{h}\right) \qquad (i=1,2,\ldots,N)$$

which represents the system of wealth-based expenditure equations for non-shelter goods. The additional term subtracted from discretionary wealth can be thought of as household wealth set

aside as a cushion to protect against the uncertainty of the future price of owned housing.

The corresponding demand equation for housing services can be written initially in the form

$$(5.12) \qquad n_0 h_{10} = n_0 \bar{h} + \left(\alpha_1^* / (\alpha_1+\beta_1+\gamma_1)\right)\left(W_0 - \bar{W}_0^* - (\lambda\sigma/(1+\rho)) n_0 \bar{h}\right)$$

where $\alpha_1^* = \alpha_1 / \left(q_0^* + \lambda\sigma/(1+\rho)\right)$. Now let V_0 stand for the value of the owned dwelling at time 0 and note that $V_0 = n_0 H_0 P_0 = n_0 \theta h_{10} P_0$. Having chosen units of measurement so that $\theta = 1$ and $P_0 = 1$, we have $V_0 = n_0 h_{10}$. Letting $\bar{V}_0$ be the current-period value of minimum required (owned) housing, we also have $\bar{V}_0 = n_0 \bar{h}$. Equation (5.12) may therefore be written in the equivalent form

$$(5.13) \qquad V_0 = \bar{V}_0 + \left(\alpha_1^* / (\alpha_1+\beta_1+\gamma_1)\right)\left(W_0 - \bar{W}_0^* - (\lambda\sigma/(1+\rho))\bar{V}_0\right)$$

Owned-housing value is thus expressed as a fraction of discretionary wealth, the latter being reduced by an amount representing the cushion or safety margin to protect against housing price uncertainty.

A third form of housing equation for owners is obtained by multiplying both sides of (5.12) by q_0^*,

$$(5.14) \qquad e_{h0}^* = \bar{e}_{h0}^* + \left(\alpha_1^{**} / (\alpha_1+\beta_1+\gamma_1)\right)\left(W_0 - \bar{W}_0^* - (\lambda\sigma/(1+\rho)) n_0 \bar{h}\right)$$

where $e_{h0}^* = n_0 h_{10} q_0^*$ and $\bar{e}_{h0}^* = n_0 \bar{h} q_0^*$ are <u>expected</u> total and minimum necessary expenditures on housing and $\alpha_1^{**} = \alpha_1 / (1+\lambda\sigma/(q_0^*(1+\rho)))$. Note that if $\lambda = 0$, then $\alpha_1^{**} = \alpha_1$, the last term in brackets on the right side of equation (5.14) vanishes, and (5.14) reduces to a form similar to (5.3). The only differences, then, are that $\bar{W}_0$ and $\bar{W}_0^*$ are defined slightly differently and that the owner equation is

interpreted in terms of expected rather than actual expenditures. If we write

$$(5.15) \qquad e_{0}^{*} = e_{h0}^{*} + \sum_{i=1}^{N} e_{i0}; \qquad \bar{e}_{0}^{*} = \bar{e}_{h0}^{*} + \sum_{i=1}^{N} \bar{e}_{i0}^{*}$$

we can then manipulate these equations along with equations (5.11) and (5.14) in such a way as to eliminate the wealth variables, as we did in the renter case. Doing so, we obtain current-period expenditure equations of the form

$$(5.16) \qquad e_{i0} = \bar{e}_{i0} + \left(\beta_{1i}/(\alpha_{1}^{**}+\beta_{1})\right)\left(e_{0}^{*}-\bar{e}_{0}^{*}\right) \qquad (i=1,2,\ldots,N)$$

$$(5.17) \qquad e_{h0}^{*} = \bar{e}_{h0}^{*} + \left(\alpha_{1}^{**}/(\alpha_{1}^{**}+\beta_{1})\right)\left(e_{0}^{*}-\bar{e}_{0}^{*}\right)$$

Finally, by summing over equations (5.11) and (5.14) we obtain the owner counterpart to the renter equation (5.9):

$$(5.18) \qquad e_{0}^{*} = \bar{e}_{0}^{*} + \left((\alpha_{1}^{**}+\beta_{1})/(\alpha_{1}+\beta_{1}+\gamma_{1})\right)\left(W_{0}-\bar{W}_{0}^{*}-(\lambda\sigma/(1+\rho))n_{0}\bar{h}\right)$$

Substituting for $n_0\bar{h}$, based on $\bar{V}_0 = n_0\bar{h}$, we can also write this equation in the alternative form

$$(5.19) \qquad e_{0}^{*} = \bar{e}_{0}^{*} + \left((\alpha_{1}^{**}+\beta_{1})/(\alpha_{1}+\beta_{1}+\gamma_{1})\right)\left(W_{0}-\bar{W}^{*}-(\lambda\sigma/(1+\rho))\bar{V}_{0}\right)$$

We have maximized the conditional utility functions of equations (4.28) and (4.29) to obtain all of the above results. If instead we assume the combined utility function of equation (4.25), we obtain results of exactly the same form, but with the additional restrictions $\gamma_1 = \gamma_2$ and $\beta_{1i} = \beta_{2i}$ $(i=1,2,\ldots,N)$.

To sum up the section, we have used utility functions defined in the previous one to derive systems of expenditure equations for renters and owners. We have shown how current expendi-

ture on individual goods may be treated either as functions of present-value wealth or as functions of total current expenditure. In the case of renters, there is no asset-price uncertainty, and the resulting equations are of standard form. In the case of owners, price uncertainty in the housing market leads to modified forms of expenditure equations in which some of present-value wealth is not allocated among goods and time periods, but in effect is set aside as a safety margin to protect the household against unpredictable fluctuations in the value of its home. An equation relating housing value to present-value wealth can also be derived from the expenditure system for owners.

6. TENURE CHOICE

We have derived expenditure and related equations for owners and renters, starting with a specified form of utility function. An additional issue of interest is the tenure choice itself: How do households decide whether to rent or own?

The basic utility framework can be used to derive a tenure-choice function. Let us assume the utility function specified in equation (4.25), according to which all households have the same set of preferences. The household maximizes U_0 under the alternatives $\eta = 1$ and $\eta = 0$, and then chooses the alternative that yields the greater maximum. For consistency with the expenditure system we may rewrite the utility function in terms of expenditure variables, prices, and the number of consuming units. To take the renter case for illustration ($\eta = 0$), the utility function may be rewritten as

$$(6.1)\quad U_0 = \alpha_2 \ln(e_{h0}-\bar{e}_{h0}) + \sum_{i=1}^{N} \beta_i \ln(e_{i0}-\bar{e}_{i0}) + \gamma \ln(a_1-\bar{a}_1) - \sum_{i=1}^{N} \alpha_2 \ln(n_0 r_0) - \sum \beta_i \ln(n_0 p_{i0})$$

Substituting for $(e_{h0}-\bar{e}_{h0})$ and $(e_{i0}-\bar{e}_{i0})$, based on an obvious adaptation of equations (5.2) and (5.3),

$$(6.2)\quad \tilde{U}_0 = \alpha_2 \ln(W_0-\bar{W}_0) + \sum_{i=1}^{N} \beta_i \ln(W_0-\bar{W}_0) + \gamma \ln(a_1-\bar{a}_1) - \alpha_2 \ln(n_0 r_0) - \sum_{i=1}^{N} \beta_i \ln(n_0 p_{i0}) + \alpha_2 \ln(\alpha_2/(\alpha_2+\beta+\gamma)) + \sum_{i=1}^{N} \beta_i \ln(\beta_i/(\alpha_2+\beta+\gamma))$$

where $\tilde{U}_0$ indicates that U_0 is evaluated at its maximum and $\beta = \sum\beta_i$. Further substitution can be performed to eliminate the term $(a_1-\bar{a}_1)$ and write the entire utility function in indirect form, in terms of wealth, price, and demographic variables, together with the relevant parameters. In general functional notation, we can express the result as

$$(6.3) \quad \tilde{U}_0 = f(W_0;\bar{W}_0;r_0;p_{i0},i=1,2,\ldots,N;n_0;k_0;\alpha_2;\beta_i,i=1,2,\ldots,N;\gamma)$$

A similar procedure can be followed in the case of owners. Letting $\tilde{U}_0^*$ be maximum utility under the owner option (η=1), we have

$$(6.4) \quad \tilde{U}_0^* = f^*(W_0;\bar{W}_0^*;q_0^*;\sigma^2;p_{i0},i=1,2,\ldots,N;n_0;k_0;\alpha_1;\beta_i,i=1,2,\ldots,N;\gamma;\lambda)$$

The difference between the utility maxima can then be expressed as a function with general form

$$(6.5) \quad \tilde{U}_0^*-\tilde{U}_0 = g(W_0;\bar{W}_0;\bar{W}_0^*;r_0;q_0^*;\sigma^2;p_{i0},i=1,2,\ldots,N;n_0;k_0;\alpha_1;\alpha_2;\beta_i,i=1,2,\ldots,N;\gamma;\lambda)$$

The function can be given a simpler appearance by noting that for purposes of subsequent estimation with cross-section survey data, all prices must necessarily be treated as constants. Thus r_0, q_0^*, and the p_{i0} are fixed, as well as σ^2 and the parameters $\alpha_1,\alpha_2,\gamma,\lambda$, and the β_i. We can then write

$$(6.6) \quad \tilde{U}_0^*-\tilde{U}_0 = g(W_0;\bar{W}_0;\bar{W}_0^*;n_0;k_0;\text{constants})$$

The actual function is quite complicated for purposes of econometric estimation but it can be approximated linearly, and in prac-

tice this approximation turns out to be quite satisfactory.

The household is assumed then to calculate $\tilde{U}_0^* - \tilde{U}_0$ and to make its tenure decision in accordance with the rules

(6.7) If $\tilde{U}_0^* - \tilde{U}_0 > 0$, then $\eta = 1$; If $\tilde{U}_0^* - \tilde{U}_0 \leq 0$, then $\eta = 0$

7. DATA

The basic data source for this study is the 1982 Canadian Family Expenditure Survey (FAMEX). This survey was carried out by Statistics Canada in February and March of 1983. All flow variables relate to the 1982 calendar year; stock-type variables (value of dwelling, balance of mortgage principal outstanding) relate to December 31 of that year.

We were asked to, and did, consider the Statistics Canada Survey of Consumer Finances as a possible source of microdata for the project, and we considered also some other microdata sources. However, we came to the conclusion that FAMEX was in fact the only source that was useful for our purposes, inasmuch as it is the only one that provides comprehensive expenditure data, coupled with income, demographic, and housing value data. It has its limitations, and we shall discuss these in section 11, but in the absence of expenditure data it is hard to see how any of the other sources could be used in the construction of integrated housing asset-expenditure models. We have used FAMEX exclusively as our source of microdata.

The 1982 FAMEX survey was based on a stratified, clustered household sample design with multistage selection probabilities. The sampling frame is of the kind used in the Statistics Canada Labour Force Survey, as is the sampling methodology generally. The survey design has associated with it variable selection probabilities and weights are therefore attached to the individual sample records. However, we have used the data in unweighted form. This practice is usually preferred in econometric work on the grounds that if the object is to estimate the parameters of a correctly

specified model, with constant coefficients, weighting the observations does not lead to any improvement in efficiency of estimation. (See especially Porter, 1973, on this point; see also DuMouchel and Duncan, 1983, on the general issue of using survey weights in multiple regression analysis.)

The unit of observation in FAMEX is referred to as the spending unit. This unit is defined as either a group of persons who are dependent on a common or pooled income for major items of expense and who are living in the same dwelling, or a financially independent individual living alone. (See the documentation provided by Statistics Canada to accompany the 1982 FAMEX public use microdata file; the public use tape containing this file is what we have been working with.) The spending unit so defined seemed appropriate for our purposes, but with some restrictions.

The total number of spending units for which information is available on the FAMEX tape is 10,938. However, we have elected to define a more homogeneous set of units, one to which our model might be thought to apply more or less uniformly. We have therefore imposed the following restrictions and filtered out all units that do not conform with them.

(1) Only units with married couples are chosen.

(2) The place of residence must be nonfarm and located in an urban area of 30,000 or more population.

(3) The husband must be in the age range 25 to 74, inclusive.

(4) Units with rent-free tenancy or with "mixed tenure" (owned for part of the year, rented for part) are excluded.

(5) Renter units with "reduced rent" are excluded.

(6) Units with dwelling values recorded as $999,999 on the tape are excluded (on the grounds that the six-digit coding makes it impossible to know the actual values of dwellings of $1,000,000 or more).

(7) Units reporting negative income before taxes are excluded.

The combined effect of these restrictions is to take the number of sample units from 10,938 down to 4,462. Of the latter total, 3,483 are owner units and 979 are renter units. The bulk of the reduction of sample size is a consequence of the age, married-couple, and urban size restrictions.

A further restriction is imposed below to eliminate renter units with relatively very large asset holdings. This restriction was adopted after initial experimental estimation indicated that a small number of high-wealth households could have a large effect on the estimated equations, a finding that is consistent with that of other studies in which Canadian microdata sets have been used for the econometric estimation of household relationships. We discuss this type of exclusion in section 9.

The 1982 FAMEX public use data tape does not include information on geographic location below the level of the five broad Canadian regions. Limited coding by rural-urban location and urban size group is provided, but individual urban centres are not identified. This makes it impossible to associate households with particular urban housing markets, and hence to incorporate market-specific characteristics, in particular interurban price differentials and differences in price variability. We regard this as one of the major deficiencies of the 1982 FAMEX data set for purposes

of modelling housing decisions at the micro-level. The matter is discussed further in section 11. Our particular choice and use of variables from the FAMEX set is discussed in section 8.

We have made some use of other data to supplement the FAMEX set. A calculation of the standard deviation of owned-housing prices is required, and for this purpose we have used time series from the Royal LePage Survey of Canadian House Prices. The Royal LePage series go back as far as 1975, although for many individual areas the series did not commence until more recently. Estimates of "fair market value" (as Royal LePage terms them) are provided for four categories of "single-family housing" and two categories of "condominium high-rise apartment." We have restricted our attention to two of the "single-family" categories, namely "detached bungalows" and "executive detached two-storey" homes. (These are the only categories for which the time series go back far enough for our purposes.) Pooling the data for these two categories, and for all areas in which figures are available, we have calculated an all-Canada estimate of σ, the standard deviation of owned-housing prices, as defined for the model. The estimated 1981, 1982, and 1983 prices for each category of housing were divided by the national Consumer Price Index to convert them to "real" values, and then "normalized" for each category in each area by dividing them by the area-category 1981-83 average price. (This was done to achieve comparability across areas and categories, and for consistency with the choice-of-units assumption in the model, which makes P_0 equal to one.) The standard deviation across all areas and both housing categories was then calculated and used as the estimate of

σ. In total, there were 741 individual annual area-category prices involved in this calculation. The estimate of σ obtained in this way is .114, implying a standard deviation of "real" housing prices in the period 1981-83 equal to 11.4 percent of the mean price. The use of this estimate in interpreting the estimated parameters of the model is discussed in section 9.

The model requires also an estimate of ρ. Our choice for this is the real after-tax rate of return on tangible assets estimated by Jenkins, which is 4.14 percent per annum. (See Jenkins, 1981, Burgess, 1981, and other references cited in those articles.) The real rate embodied in our model should be interpreted as an expected or ex ante rate. (Actual ex post rates of return fluctuate from year to year.) A long-run rate such as that estimated by Jenkins is appropriate for our purposes.

The calculation of q_0^* for the model requires an estimate of d_0, the operating cost of an owned dwelling, including depreciation. We have included in our definition of d_0 all components of shelter costs reported in FAMEX for owners, with the exception of mortgage interest (which is better viewed as a cost of borrowing than a current operating cost, for our purposes) and "other accommodation" (which includes accommodation while away from home, and clearly should be omitted). This means that we have included "maintenance, repairs, and replacements," "property taxes," "homeowners' insurance premiums," and "water, fuel, and electricity." The definition of rent is set correspondingly, and includes reported rent plus expenses for water, fuel, and electricity. The inclusion of water, fuel, and electricity costs is based on a judgement by us that it would be difficult to be sure that reported

rent did not include some components of these costs in individual cases, and therefore difficult to establish consistency in reported rent from renter household to renter household and consistency between shelter costs for renters and shelter costs for owners, unless such costs were uniformly included. Another argument for their inclusion is that if water, fuel, and electricity are used in fixed ratios to size of dwelling, they cannot be identified as independently variable arguments in the utility function (i.e., as belonging to the set of x-variables) and should therefore be merged with the housing variables. In practice, there will not be a fixed proportionality relationship with housing but the assumption of an approximate relationship of this kind may be considered adequate for our purposes.

The depreciation component of d_0 cannot be determined for individual households from a cross-section data set. One household may repair its roof in a given year; another may replace a worn-out furnace motor; a third may paint its house; and a fourth may have no repair, replacement, or maintenance costs at all in the given year. We would therefore expect the "maintenance, repairs, and replacements" component of individual expenditures to be highly variable from household to household in the survey year, but to average out to a meaningful rate of expenditure over all owner households. What we have done, therefore, is to calculate a single value for d_0 by taking the ratio of total housing operating costs (as we define them) over all owner households in our data set, to the corresponding total value of all homes owned. The resulting value of d_0 is .0431, which is interpreted as the annual operating

cost per unit of owned dwelling. The annual expected full price of living in an owned dwelling (including both operating cost and the opportunity cost of funds) is then given by $q_0^* = d_0 + \rho/(1+\rho) =$ .0431 + (.0414/1.0414) = .0829. This may be interpreted as the user cost per dollar of housing asset.

8. AN EXPENDITURE-TENURE MODEL THAT CAN BE ESTIMATED WITH AVAILABLE DATA

The basic expenditure system derived within the modified Stone-Geary utility framework can be put into several alternative forms, as shown in section 5. The form that we have considered most suitable as a basis for defining an econometric system to be estimated with the cross-section micro-data provided on the 1982 FAMEX public use tape consists of equations (5.7), (5.8), and (5.9), for renters, and equations (5.16), (5.17), and (5.18), for owners. In addition, we incorporate into the system a tenure-choice equation of the general form of equation (6.6). Adding stochastic error terms to these equations, the theoretical system to be estimated, expressed in its simplest form, is then as follows:

Renters

$$(8.1) \quad e_0 = \bar{e}_0 + \xi(W_0 - \bar{W}_0) + u_0$$

$$(8.2) \quad e_{h0} = \bar{e}_{h0} + \pi_h(e_0 - \bar{e}_0) + u_{h0}$$

$$(8.3) \quad e_{i0} = \bar{e}_{i0} + \pi_i(e_0 - \bar{e}_0) + u_{i0} \qquad (i=1,2,\ldots,N)$$

Owners

$$(8.4) \quad e_0^* = \bar{e}_0^* + \xi^*\left(W_0 - \bar{W}_0^* - (\lambda\sigma/(1+\rho))n_0\bar{h}\right) + u_0^*$$

$$(8.5) \quad e_{h0}^* = \bar{e}_{h0}^* + \pi_h^*\left(e_0^* - \bar{e}_0^*\right) + u_{h0}^*$$

$$(8.6) \quad e_{i0} = \bar{e}_{i0} + \pi_i^*\left(e_0^* - \bar{e}_0^*\right) + u_{i0}^* \qquad (i=1,2,\ldots,N)$$

Tenure choice

$$(8.7) \quad \tilde{U}_0^* - \tilde{U}_0 = g\left(W_0; \bar{W}_0; \bar{W}_0^*; n_0; k_0; \text{constants}\right) + \bar{u}_0$$

where $\xi = (\alpha_2+\beta_2)/(\alpha_2+\beta_2+\gamma_2)$

$\pi_h = \alpha_2/(\alpha_2+\beta_2)$

$$\pi_i = \beta_{2i}/(\alpha_2+\beta_2) \qquad (i=1,2,\ldots,N)$$

$$\xi^* = (\alpha_1^{**}+\beta_1)/(\alpha_1+\beta_1+\gamma_1)$$

$$\pi_h^* = \alpha_1^{**}/(\alpha_1^{**}+\beta_1)$$

$$\pi_i^* = \beta_{1i}/(\alpha_1^{**}+\beta_1) \qquad (i=1,2,\ldots,N)$$

and $\beta_1 = \sum_{i=1}^{N} \beta_{1i}$, $\beta_2 = \sum_{i=1}^{N} \beta_{2i}$, as before.

We interpret this system of equations in the following way. An individual household first makes a tenure decision, based on equation (8.7): if the value of the tenure-choice function is positive, the household chooses to own; if the value is nonpositive, it chooses to rent. Conditional on choosing to rent, the household then allocates expenditures according to equations (8.1)-(8.3); conditional on choosing to own, it allocates according to equations (8.4)-(8.6). Whatever its tenure choice, it makes its subsequent expenditure decisions in two stages: (1) using either (8.1) or (8.4), it chooses its current level of expenditure (aside from a stochastic error), based on its calculated lifetime wealth and (in the case of owners) the uncertainty about the future price of housing; (2) using either (8.2) and (8.3), or (8.5) and (8.6), it allocates its current expenditure total among shelter and non-shelter expenditures. (By "expenditure" we mean "expected expenditure" in the case of total and shelter expenditures for owners, since these include a random housing-price component.) The stochastic errors in the equations (the u-variables) are assumed to be independently and normally distributed, with zero means and constant variances. However, they are subject to the restrictions

$$(8.8) \qquad u_{h0} + \sum_{i=1}^{N} u_{i0} = 0$$

$$(8.9) \qquad u^*_{h0} + \sum_{i=1}^{N} u^*_{i0} = 0$$

In addition, the coefficients of equations (8.2) and (8.3), and of (8.5) and (8.6), are subject to the restrictions

$$(8.10) \qquad \pi_h + \sum_{i=1}^{N} \pi_i = 0$$

$$(8.11) \qquad \pi^*_h + \sum_{i=1}^{N} \pi^*_i = 0$$

An implication of (8.8)-(8.11) is that only N of the N+1 equations defined by (8.2) and (8.3), or by (8.5) and (8.6), are independent; given any N of the N+1 equations in each of these subsets, the other equation in the subset is automatically determined.

We now adapt the theoretical system for econometric estimation with the available data. Considering first equation (8.2), for illustration, we recognize that π_h may vary with age, and we recognize also that $\bar{e}_{h0}$ and $\bar{e}_0$ will be related to the size and age composition of the household. In the notation that we have adopted in specifying the econometric equations, we write

$$(8.12) \qquad \pi_h = \sum_{j=1}^{5} \pi_{hj} A_j$$

$$(8.13) \qquad \bar{e}_{h0} = \sum_{k=1}^{4} \varphi_{hk} D_k$$

$$(8.14) \qquad \bar{e}_0 = \sum_{k=1}^{4} \theta_k D_k$$

where A_1 = 1 if husband is aged 25-34, 0 otherwise

A_2 = 1 if husband is aged 35-44, 0 otherwise

A_3 = 1 if husband is aged 45-54, 0 otherwise

A_4 = 1 if husband is aged 55-64, 0 otherwise

A_5 = 1 if husband is aged 65-74, 0 otherwise

D_1 = number of person-years for persons aged 0-6

D_2 = number of person-years for persons aged 7-12

D_3 = number of person-years for persons aged 13-17

D_4 = number of person-years for persons aged 18 or over

Thus five age categories are recognized for age of husband (representing his own age group and that of his wife) and four age categories for the demographic composition of the household. FAMEX reports the numbers of person-years in the household in 1982 for persons in each of eight age groups, thereby taking account of situations in which the composition of the household changed over the course of the year. It also reports the actual age of the husband. However, we have considered it desirable for our purposes to work with the combinations indicated above.

Inserting (8.12)-(8.14) into (8.2), we obtain the initial econometric specification of the shelter equation for renters:

$$(8.15) \qquad e_{h0} = \sum_{k=1}^{4} \varphi_{hk} D_k + \Big(\sum_{j=1}^{5} \pi_{hj} A_j \Big) \Big(e_0 - \sum_{k=1}^{4} \theta_k D_k \Big) + u_{h0}$$

Rearranging the right side of the equation, we then obtain the alternative form:

$$(8.16) \qquad e_{h0} = \sum_{k=1}^{4} \varphi_{hk} D_k + \sum_{j=1}^{5} \pi_{hj} A_j e_0 - \sum_{j=1}^{5} \sum_{k=1}^{4} \pi_{hj} \theta_k A_j D_k + u_{h0}$$

The shelter expenditure equation is thus seen to be linear in the variables D_k, $A_j e_0$, and $A_j D_k$, but nonlinear in the π and θ parameters. In fact, the variables on the right side of the equation are subject to four linear restrictions that make it impossible to estimate the equation in the form specified:

$$(8.17) \qquad D_k = \sum_{j=1}^{5} A_j D_k = D_k \sum_{j=1}^{5} A_j \qquad (k=1,2,3,4)$$

(Note that the A_j variables sum to unity.) As a practical matter, this requires that four of the 24 variables in the set (D_k, $A_j D_k$; k=1,2,3,4; j=1,2,3,4,5) be dropped. To satisfy this requirement, we omit the four D_k variables and rewrite equation (8.16) as

$$(8.18) \qquad e_{h0} = \sum_{j=1}^{5} \pi_{hj} A_j e_0 - \sum_{j=1}^{5} \sum_{k=1}^{4} \delta_{hjk} A_j D_k + u_{h0}$$

where $\delta_{hjk} = \pi_{hj} \theta_k$. The δ parameters are implicitly subject to nonlinear restrictions implied by equation (8.16) coupled with (8.17). However, even with the restrictions taken account of, it turns out to be impossible to identify the individual θ and φ parameters of (8.16), and hence the $\bar{e}_{h0}$ and $\bar{e}_0$ terms in the original theoretical equation (8.2). The reason for the lack of identifiability is essentially the absence of price variables. The estimation of linear expenditure systems with time-series data, or with richer sets of cross-section data, may permit the incorporation of price variables, and hence the identifiability of the minimum required expenditure parameters. Unfortunately that is not the case here. Lack of identifiability implies that in our case

these parameters cannot be estimated by any statistical procedure.

The π parameters of equation (8.18) are identifiable; they are simply the coefficients of the $A_j e_0$ interaction variables, and can be estimated directly. Moreover, consistent estimators of the π parameters (in the theoretical statistical sense) can be obtained by "ordinary" least squares (OLS), without imposition of any constraints. (Roughly speaking, if the model is correctly specified and the parameter restrictions are also correct, with a large enough sample size the restrictions will be satisfied anyway, even if they are not imposed explicitly.) We have chosen therefore to estimate the π parameters by applying OLS to equation (8.18).

Equations (8.3), (8.5), and (8.6) can be put into forms similar to that of equation (8.18) and the corresponding π parameters estimated in the same way. However, a more complicated procedure is required to convert equations (8.1) and (8.4) into a form that can be estimated. The major difficulties are (1) that the FAMEX data set provides only limited information about household assets and liabilities, and (2) that those components of wealth that represent present values of future income or expenditure streams, or future demographic composition of the household, are intrinsically impossible to measure by any direct method.

The FAMEX data set does provide information on household income (by type of income), on the value of owner-occupied dwellings, and on the amount of mortgage loan principal outstanding. (Dwelling value and principal outstanding are reported as at December 31, 1982, rather than at the middle of 1982, as required

under the assumptions of the theoretical model, strictly interpreted. However, as a practical matter this inconsistency is of negligible importance.) What we have done, as a first step, is to calculate an asset value for each household, based on the household's homeowner equity (dwelling value minus mortgage principal outstanding) and on an imputed value of nonhousing assets. Asset value times rate of return is equal to investment income, by definition. Inverting this identity, investment income divided by rate of return is equal to asset value. If Y^* is investment income and r^* is the rate of return (the nominal rate, for this purpose), we then calculate the imputed value of nonhousing assets as Y^*/r^*. We assume, for the calculation, that r^* is equal to the average yield on six-month treasury bills in 1982, which was 13.75 percent. Based on this imputation, we define total asset value as

$$(8.19) \qquad Z = V-M+(Y^*/r^*)$$

where $r^* = .1375$, V is dwelling value, and M is mortgage principal outstanding. Note that $Z = Y^*/r^*$ for renters.

The next problem is that of how to represent the present-value wealth associated with future income streams. Future income includes wage and salary payments, retirement pensions based on employment, transfer payments of various kinds, and other items. Some of these income flows are associated with remunerative employment and some are not. (Wages and salaries are obviously employment-related, whereas Old Age Security payments depend only on the attainment of a given age.) We assume then that present value "human wealth" can be regarded as a function of observed current income (representing in part the current return on the

human wealth) and the present age of the household, as represented by the husband's age. (In addition to other effects, husband's age reflects the length of the future income stream and of the discounting period.) The present value of future minimum necessary wealth is related to future household demographic characteristics; these are unobservable, but we assume that they can be treated as functions of observed present demographic composition and husband's age. The term $(\lambda\sigma/(1+\rho))n_0\bar{h}$ in the owner-household total expenditure equation, (8.4), is also a function of present demographic composition, as embodied in n_0. Putting all of this together, we proxy the wealth-related variables in equations (8.1) and (8.4) as follows:

$$(8.20)\qquad W_0-\bar{W}_0 = Z + \sum_{j=1}^{5} b_j A_j + \sum_{j=1}^{5} c_j A_j (Y-Y^*) + \sum_{j=1}^{5}\sum_{k=1}^{4} d_{jk} A_j D_k$$

$$(8.21)\qquad W_0-\bar{W}_0^* - (\lambda\sigma/(1+\rho))n_0\bar{h} = Z + \sum_{j=1}^{5} b_j^* A_j + \sum_{j=1}^{5} c_j^* A_j (Y-Y^*)$$

$$+ \sum_{j=1}^{5}\sum_{k=1}^{5} d_{jk}^* A_j D_k$$

where Y is total reported income before tax (so that $Y-Y^*$ is before-tax noninvestment income) and the other variables are as defined previously. Note that in doing its present-value calculations, the household is assumed to have in mind a set of values for all future (real) incomes and an after-tax interest or discount rate. The discounting effects at different ages are assumed to be caught up in the coefficients, which are allowed to vary with age. Differences in length of discounting period are embod-

ied in these coefficients, as are the effects of income taxation.

We now substitute for the wealth-related variables in equations (8.1) and (8.4) based on equations (8.20) and (8.21), for $\bar{e}_0$ based on equation (8.14), and for $\bar{e}_0^*$ based on an obvious adaptation of the latter equation. In addition, we assume that the ξ and ξ^* parameters of (8.1) and (8.4) vary with age of husband (as the theory suggests). This yields

$$(8.22) \quad e_0 = \sum_{k=1}^{4} \theta_k D_k + \Big(\sum_{j=1}^{5} \xi_j A_j \Big)\Big(Z + \sum_{j=1}^{5} b_j A_j + \sum_{j=1}^{5} c_j A_j (Y-Y^*) + \sum_{j=1}^{5} \sum_{k=1}^{4} d_{jk} A_j D_k \Big) + u_0$$

$$(8.23) \quad e_0^* = \sum_{k=1}^{4} \theta_k^* D_k + \Big(\sum_{j=1}^{5} \xi_j^* A_j \Big)\Big(Z + \sum_{j=1}^{5} b_j^* A_j + \sum_{j=1}^{5} c_j^* (Y-Y^*) + \sum_{j=1}^{5} \sum_{k=1}^{4} d_{jk}^* A_j D_k \Big) + u_0^*$$

Rearranging the right sides of these equations, and noting that $A_i A_j$ is equal to 1 for $i=j$ and 0 for $i \neq j$, we obtain

$$(8.24) \quad e_0 = \sum_{k=1}^{4} \theta_k D_k + \sum_{j=1}^{5} \xi_j A_j Z + \sum_{j=1}^{5} \xi_j b_j A_j + \sum_{j=1}^{5} \xi_j c_j A_j (Y-Y^*) + \sum_{j=1}^{5} \sum_{k=1}^{4} \xi_j d_{jk} A_j D_k + u_0$$

$$(8.25) \quad e_O^* = \sum_{k=1}^{4} \theta_k^* D_k + \sum_{j=1}^{5} \xi_j^* A_j Z + \sum_{j=1}^{5} \xi_j^* b_j^* A_j + \sum_{j=1}^{5} \xi_j^* c_j^* A_j (Y-Y^*)$$

$$+ \sum_{j=1}^{5} \sum_{k=1}^{4} \xi_j^* d_{jk}^* A_j D_k + u_O$$

The parameters of these equations are subject to a large number of nonlinear constraints. However, the ξ and ξ^* parameters, which are the ones of principal interest, are simply the coefficients of the A_jZ interaction variables. As before, they can be estimated consistently by OLS (under standard assumptions about the error terms), and this is again the strategy that we adopt in the case of owners; as before, we ignore the restrictions on the parameters. However, in the case of renters we adopt a somewhat different strategy. The renter sample is much smaller than the owner sample and, in addition, tangible assets are (on average) a much smaller proportion of total present-value wealth for renters than for owners. In consequence (and as our early experimental estimation indicated), the ξ parameters are much less well determined than the ξ^* parameters when constraints are ignored. Our strategy, therefore, is to use the unrestricted owners equation to estimate the ratios of the coefficients of the A_jZ variables to the coefficients of the corresponding $A_j(Y-Y^*)$ variables, and then impose these ratios as proportionality restrictions on the coefficients of the renter equation. The validity of this procedure stems from the theoretical requirement that $c_j=c_j^*$ $(j=1,\ldots,5)$, since both sets of coefficients represent the conversion of noninvestment income into present-value wealth, and should

be unrelated to tenure status. The renter equation is thus estimated by restricted least squares (RLS).

The D_k and A_jD_k variables are subject to a set of exact linear relations, as before, and cannot all be included in the equations for estimation purposes; we therefore drop the terms involving the D_k variables alone. The final forms of the equations specified for estimation are thus

$$(8.26) \quad e_O = \sum_{j=1}^{5} \xi_j A_j Z + \sum_{j=1}^{5} \bar{b}_j A_j + \sum_{j=1}^{5} \bar{c}_j A_j (Y-Y^*) + \sum_{j=1}^{5} \sum_{k=1}^{4} \bar{d}_{jk} A_j D_k + u_O$$

$$(8.27) \quad e_O^* = \sum_{j=1}^{5} \xi_j^* A_j Z + \sum_{j=1}^{5} \bar{b}_j^* A_j + \sum_{j=1}^{5} \bar{c}_j^* A_j (Y-Y^*) + \sum_{j=1}^{5} \sum_{k=1}^{4} \bar{d}_{jk}^* A_j D_k + u_O$$

Each of these equations involves 35 independent variable terms, and 35 associated parameters. As noted, equation (8.27) is estimated by OLS while (8.26) is estimated by RLS, under the restrictions $\bar{c}_j = \left(\bar{c}_j^*/\xi_j^*\right)\xi_j$ (j=1,...,5). (Note that the equations contain no intercept terms; the A_j variables sum to unity, and an independent intercept term cannot therefore be included.) It may be noted that, as with the earlier equations, it is not possible to identify the basic theoretical parameters associated with minimum consumption or wealth requirements.

The last of the equations of the model defined by (8.1)-(8.7) to be considered for estimation purposes is the tenure-

choice equation. As written in (8.7), this equation is free of any particular functional form. For estimation purposes, we assume a linear approximation.

The theory underlying equation (8.7) suggests that the set of independent variables should be similar to those that enter equations (8.1) and (8.4) in the theoretical model, or (8.26) and (8.27) in the forms specified for estimation. In other words, tenure choice should be a function of wealth and demographic variables (with price variables fixed, and hence treated as constants); with the substitutions indicated above, this means that tenure choice becomes a function of assets, current noninvestment income, husband's age, and demographic composition variables. In addition, we assume that tenure preference is related to the presence of children in the household, as distinct from the numbers of children in different age groups: the presence of one or more children under 18 years of age is expected to tilt the parents' preferences in favour of home ownership. We therefore incorporate a variable C defined by

C = 1 if the household contains any children under the age of 18, 0 otherwise

Assuming the error term in (8.7) to have a standard normal distribution, the tenure choice function can be specified as a probit function. The probabilities of owning (η=1) and not owning (η=0) are given by

$$(8.28) \quad Pr(\eta=1) = Pr(\tilde{U}_O^* - \tilde{U}_O > 0); \quad Pr(\eta=0) = Pr(\tilde{U}_O^* - \tilde{U}_O^* \leq 0)$$

and the difference between the utility maxima under the two op-

tions is given by

$$(8.29)\qquad \tilde{U}_0^* - \tilde{U}_0 = g\left(C; A_j; A_j Z; A_j(Y-Y^*); D_k; j=1,2,3,4,5; k=1,2,3,4\right) + \bar{u}_0$$

The probit function of equation (8.29) is specified as linear in the 20 variables indicated. We estimate its parameters by a maximum likelihood procedure.

9. PARAMETER ESTIMATES DERIVED FROM THE EXPENDITURE EQUATIONS

We have estimated a system of equations corresponding to the theoretical system (8.1)-(8.7), and to the econometric specification of these equations described in the previous section. Equations were estimated (separately for renters and owners) for expenditures on food, clothing, and all other nonshelter categories combined, as well as for total shelter and nonshelter expenditures and tenure choice. In total, then, the system consists of thirteen equations. These equations are reported in full detail in the Appendix. Parameters or parameter functions of special interest derived from the expenditure equations are reported in Tables 1, 2, and 3 of the main body of the report, and selected tenure probabilities are reported in Table 4. The expenditure-equation results are discussed in the present section and the probability calculations in the next one.

The choice of food, clothing, and other nonshelter categories combined is arbitrary. We have chosen these three categories principally for illustrative purposes. The issues central to the project are unrelated to any particular breakdown of nonshelter expenditures; they can be treated entirely on the basis of equations for shelter and total nonshelter expenditures. However, we thought it might be helpful to see how our approach to the modelling of housing as consumption and investment could be applied within the context of a more comprehensive expenditure system. Obviously the system could be extended to incorporate equations for additional nonshelter categories, in as much detail as might be desired or available data would permit.

As noted previously, one of the expenditure-category equa-

tions in each of the renter and owner sets is redundant; it can be calculated residually. However, for convenience we have included in the Appendix tables estimated equations for all categories. The redundancy arises from the adding-up restriction that requires the categories to sum to total expenditure. Because the category equations are linear and have identical sets of independent variables, this adding-up constraint is preserved by least-squares estimation, or indeed by the application of any linear estimation procedure. (See Denton, 1978, for a general treatment of this issue.)

In addition to the restrictions on the data set described in section 7, we have eliminated a few "outlier" observations from the subset for renters. We observed in early analysis that some of the results for renters were quite sensitive to the inclusion of a very small number of relatively very wealthy renter households, and that with these households omitted the coefficient patterns across age groups in the estimated equations were more regular, and apparently more realistic. Accordingly, we eliminateo all renter households with assets (Z) of $100,000 or more. In practice, this meant the elimination of only eight households, but the improvement in the estimated parameter patterns was substantial. With these three households removed, there were then 971 renter households, rather than 979, as in our original renter subset.

It may be noted that others have found also that parameter estimates in cross-section household studies can be quite sensitive to the inclusion of high-wealth units. Such a finding is reported, for example, by Burbidge and Robb (1985), in connection

with their econometric study of wealth-age profiles.

Table 1 displays the estimated coefficients of the $A_j e_0$ variables in the expenditure-category equations for renters and the $A_j e_0^*$ variables in the corresponding equations for owners, together with the symbolic representation of these coefficients in terms of the basic parameters of the theoretical model. (The complete equations from which the coefficients are taken are reported in Apendix Tables A2-A6 for renters and A8-A12 for owners.) The interpretation of the coefficients in Table 1 is as the marginal shares of total expenditure (and of discretionary expenditure) that go to the individual component categories. For example, the estimated coefficients for shelter and nonshelter expenditures for renter households with husband aged 25-34 are .1116 and .8884, respectively. These figures imply that of an increase of $100 in total expenditure, $11.16 would be allocated to shelter and $88.84 to nonshelter. The proportionate shelter and nonshelter shares necessarily sum to unity and the shares for components of nonshelter (food, clothing, other) necessarily sum to the overall nonshelter shares (aside from rounding discrepancies).

Total expenditure for owner households, as we define it, is in fact <u>expected</u> total expenditure, since it is calculated as $q_0^* v_0$, and q_0^* is the <u>expected</u> annual user cost of living in an owned dwelling. (The <u>actual</u> user cost cannot be known until after the fact since it depends on what happens to housing prices over the course of the year.)

The main point to note in Table 1 is that the marginal shares of expenditure going to shelter are much higher for owners than for renters. Using owners' shelter expenditures as actually

reported in the FAMEX survey would produce lower marginal shares because the reported amounts do not include the opportunity cost of home ownership. The estimates that we have calculated on the user-cost basis no doubt give a more realistic picture.

The marginal shares going to nonshelter are much lower for owners than for renters, being the obverse of the shelter shares. For most age groups the greatest differences occur in the "other" category of nonshelter, although for the 55-64 group there is a large difference in the food expenditure share.

Table 2 provides results relating to the marginal shares of current and future consumption in present-value wealth. These results are based on the equations in Appendix Tables A1 and A7. The parametric representations of the shares are also shown. In the case of renters, all wealth is allocated (in the minds of the household planners) to consumption, either present or future. In the case of owners, though, there is a share that does not represent consumption, but rather is set aside as protection against the holding of the risky housing asset. We term this component a "risk allocation."

The risk allocation exists because of the difference between α_1 and α_1^{**}. If σ^2, the variance of the (real) price of housing, is zero, there is no risk, α_1^{**} is then equal to α_1, and the risk allocation component disappears. If $\sigma^2>0$ but households make no provision for uncertainty, then λ will be zero, and again α_1^{**} will be equal to α_1; the $\lambda=0$ case we might think of as implying risk neutrality on the part of households. In general, we would expect households to be risk averse, and hence $\lambda>0$.

The calculation of the risk allocation component requires the specification of some additional structure for the model. In particular, we stipulate that owners and renters may differ in their relative preferences for shelter and nonshelter goods but that, aside from uncertainty, their preferences between present consumption and future consumption are the same. In terms of the underlying utility-function parameters, this translates into the restriction

$$(9.1) \quad \gamma_1/(\alpha_1+\beta_1+\gamma_1) = \gamma_2/(\alpha_2+\beta_2+\gamma_2)$$

In short, if housing were a riskless asset, owners would allocate anticipated marginal present-value wealth to present consumption and future consumption in the same proportions as renters.

The total expenditure regression equation for owners allows us to estimate $(\alpha_1^{**}+\beta_1)/(\alpha_1+\beta_1+\gamma_1)$ directly, as the coefficient of Z. Equation (9.1) allows us to infer $\gamma_1/(\alpha_1+\beta_1+\gamma_1)$ from the total expenditure equation for renters, and to calculate $(\alpha_1-\alpha_1^{**})/(\alpha_1+\beta_1+\gamma_1)$ by subtraction (since the three shares must sum to unity). From the definition of α_1^{**} in the theoretical model and the values assigned to ρ, σ, and q_0^*, as stated in section 7, we can then calculate the implied values of λ shown in Table 2. Given λ and the assumption of normality for the random variation of housing asset prices, we can also calculate $F(\lambda)$, the value of the standard normal cumulative distribution function corresponding to λ. (As discussed in section 4, $F(\lambda)$ is interpreted as the probability that $\hat{a}_1$ will be above the minimum required level $\bar{a}_1$.) A risk-neutral household would have $\lambda=0$ and $F(\lambda)=.5$. As reported in the table, the values of λ are all posi-

tive and the values of $F(\lambda)$ are greater than .5, implying a preference for risk avoidance. The values of λ vary over the age groups; in particular, the low value for the 55-64 group is somewhat inconsistent with the values for other age groups but it is doubtful that much importance should be attached to this result, given the stochastic nature of the underlying sample and of the derived parameter estimates.

Estimation of λ requires the restriction represented by equation (9.1). An alternative to the use of this restriction on the model is simply to calculate what an owner's risk allocation would be under specific assumptions about the risk probability $F(\lambda)$ (and the corresponding λ value). Table 3 presents the results of some calculations under four alternative assumptions: $F(\lambda)=.5$ (risk neutrality); $F(\lambda)=.75$; $F(\lambda)=.9$; and $F(\lambda)=.99$. The first line of the table represents the marginal expenditure proportions allocated to current consumption and the second line represents the remainders. The distributions of these remainders between future consumption and risk allocation are provided in the subsequent lines of the table, under the four alternative assumptions. The calculated risk allocations in Table 3 range from 0, for $F(\lambda)=.5$ (all age groups) to .054, for $F(\lambda)=.99$ and age group 45-54. The corresponding λ values range from 0 to about 2.33. These ranges compare with the estimated ranges reported in Table 2 of .004 to .034 for the risk allocation and .20 to 1.46 for λ.

TABLE 1: ESTIMATED MARGINAL SHARES OF CURRENT EXPENDITURE ALLOCATED TO SHELTER AND NONSHELTER CATEGORIES: RENTERS AND OWNERS, BY HOUSEHOLD AGE GROUP

	Parametric representation	Household age group (age of husband)				
		25-34	35-44	45-54	55-64	65-74
Renters						
Shelter	$\alpha_2/(\alpha_2+\beta_2)$	.1116	.1265	.1554	.1630	.1604
Nonshelter	$\beta_2/(\alpha_2+\beta_2)$	.8884	.8735	.8446	.8370	.8396
Food	$\beta_{21}/(\alpha_2+\beta_2)$	.1148	.1234	.0973	.1275	.0982
Clothing	$\beta_{22}/(\alpha_2+\beta_2)$	.1148	.0968	.0968	.0976	.0694
Other	$\beta_{23}/(\alpha_2+\beta_2)$	.6588	.6533	.6505	.6119	.6720
Owners						
Shelter	$\alpha_1^{**}/(\alpha_1^{**}+\beta_1)$	.2031	.2043	.2493	.2432	.2750
Nonshelter	$\beta_1/(\alpha_1^{**}+\beta_1)$	.7969	.7957	.7507	.7568	.7250
Food	$\beta_{11}/(\alpha_1^{**}+\beta_1)$	.1009	.0876	.0956	.0808	.0871
Clothing	$\beta_{12}/(\alpha_1^{**}+\beta_1)$	.0935	.0985	.0842	.0869	.0603
Other	$\beta_{13}/(\alpha_1^{**}+\beta_1)$	.6025	.6095	.5708	.5891	.5776

TABLE 2: ESTIMATED MARGINAL SHARES OF PRESENT-VALUE ANTICIPATED WEALTH ALLOCATED TO CURRENT CONSUMPTION, FUTURE CONSUMPTION, AND RISK PROTECTION, TOGETHER WITH RISK PARAMETERS AND PROBABILITIES: RENTERS AND OWNERS, BY HOUSEHOLD AGE GROUP

	Parametric representation	Household age group (age of husband) 25-34	35-44	45-54	55-64	65-74
Renters						
Current consumption	$(\alpha_2+\beta_2)/(\alpha_2+\beta_2+\gamma_2)$	.0736	.0875	.1043	.0656	.0687
Future consumption	$\gamma_2/(\alpha_2+\beta_2+\gamma_2)$	.9264	.9125	.8957	.9344	.9313
Risk allocation	--	--	--	--	--	--
Owners						
Current consumption	$(\alpha_1^{**}+\beta_1)/(\alpha_1+\beta_1+\gamma_1)$	.0634	.0710	.0705	.0617	.0473
Future consumption	$\gamma_1/(\alpha_1+\beta_1+\gamma_1)$	.9264	.9125	.8957	.9344	.9313
Risk allocation	$(\alpha_1-\alpha_1^{**})/(\alpha_1+\beta_1+\gamma_1)$	.0102	.0165	.0338	.0039	.0214
Risk parameter	λ	.60	.86	1.46	.20	1.25
Risk probability	$F(\lambda)$	.7258	.8051	.9279	.5793	.8944

NOTE: The calculations in this table are based on the restriction $\gamma_1/(\alpha_1+\beta_1+\gamma_1) = \gamma_2/(\alpha_2+\beta_2+\gamma_2)$.

TABLE 3: ESTIMATED MARGINAL SHARES OF PRESENT-VALUE ANTICIPATED WEALTH ALLOCATED TO CURRENT CONSUMPTION, FUTURE CONSUMPTION, AND RISK PROTECTION, UNDER ALTERNATIVE ASSUMPTIONS ABOUT RISK PROBABILITIES: OWNERS, BY HOUSEHOLD AGE GROUP

	Parametric representation	Household age group (age of husband)				
		25-34	35-44	45-54	55-64	65-74
Allocated to current consumption	$(\alpha_1^{**}+\beta_1)/(\alpha_1+\beta_1+\gamma_1)$	.0634	.0710	.0705	.0617	.0473
Not allocated to current consumption	$(\gamma_1+\alpha_1-\alpha_1^{**})/(\alpha_1+\beta_1+\gamma_1)$	.9366	.9290	.9295	.9383	.9527
If F(λ)=.5 (λ=0):						
Future consumption	$\gamma_1/(\alpha_1+\beta_1+\gamma_1)$	.9366	.9290	.9295	.9383	.9527
Risk allocation	$(\alpha_1-\alpha_1^{**})/(\alpha_1+\beta_1+\gamma_1)$	.0000	.0000	.0000	.0000	.0000
If F(λ)=.75 (λ=.67):						
Future consumption	$\gamma_1/(\alpha_1+\beta_1+\gamma_1)$	.9252	.9162	.9140	.9250	.9412
Risk allocation	$(\alpha_1-\alpha_1^{**})/(\alpha_1+\beta_1+\gamma_1)$	.0114	.0128	.0155	.0133	.0115
If F(λ)=.9 (λ=1.28):						
Future consumption	$\gamma_1/(\alpha_1+\beta_1+\gamma_1)$	.9148	.9045	.8998	.9129	.9307
Risk allocation	$(\alpha_1-\alpha_1^{**})/(\alpha_1+\beta_1+\gamma_1)$	.0218	.0245	.0297	.0254	.0220
If F(λ)=.99 (λ=2.33):						
Future consumption	$\gamma_1/(\alpha_1+\beta_1+\gamma_1)$	.8970	.8844	.8754	.8921	.9127
Risk allocation	$(\alpha_1-\alpha_1^{**})/(\alpha_1+\beta_1+\gamma_1)$	.0396	.0446	.0541	.0462	.0400

10. SOME TENURE PROBABILITY CALCULATIONS

The tenure-choice function, as reported in Appendix Table A13, is used now to provide some illustrative calculations of the probabilities of owning a home. These probabilties are presented in Table 4. Separate calculations are provided for each of the five household age groups and for selected characteristics within those age groups.

The initial probability calculations shown in the table are based on average values of assets, noninvestment income, and demographic-composition variables within each age group. The averages, which are computed from the data set that we have used to estimate the model, are denoted by $\bar{Z}$, $\bar{Y}$-$\bar{Y}^*$, $\bar{D}_1$, $\bar{D}_2$, $\bar{D}_3$, $\bar{D}_4$, and $\bar{C}$. In the case of $\bar{Z}$ and $\bar{Y}$-$\bar{Y}^*$, the averages are represented by sample median values; in the case of the other variables, they are represented by sample means. Medians are used in the first two cases to avoid undue influence on the results by households with very high asset and income levels. (Initial calculations indicated that such influence could be a problem if means were used for $\bar{Z}$ and $\bar{Y}$-$\bar{Y}^*$.) Means are used for the demographic-composition variables to give a more balanced representation of the age distributions of household members than would be provided by the medians. (The median household with husband 25-34, for example, has exactly one child aged 0-6 and no other children; the mean household, on the other hand, has .842 children aged 0-6, .303 aged 7-12, and .042 aged 13-17.) The average household characteristics are shown in the top part of Table 4 and the probability calculations in the lower half. The cases chosen for illustration include households with average characteristics, with assets and incomes 50 percent above and 50 per-

cent below the averages, with numbers of children 50 percent above and below, and with two adults and no children.

The dominance of affluence and household age in determining tenure are very clear from Table 4. Households with average characteristics are almost certain to own their own homes from age 35 on. Even households with asset and income levels 50 percent below the average have high probabilities of being owners, although the probabilities are markedly lower for such households before the age of 35, and again after the age of 65. Households with larger numbers of children have somewhat higher ownership probabilities and households with fewer children have somewhat lower probabilities before the age of 35, but thereafter there are hardly any differences. Two-adult households (no children at any time over the life cycle) have about a 50/50 chance of owning under age 35, but they too are almost certain to own when they are beyond that age.

It should be stressed that the sample on which these calculations are based is one with specially chosen characteristics. As described in section 7, it includes only households in large urban centres with both husband and wife present. The ownership probabilities would undoubtedly be lower for unattached individuals and single-parent households. For husband-wife households with the characteristics that we have prescribed, the calculations serve to highlight the fact that renting is associated very largely with youth and relatively low standing in the asset and income distributions.

TABLE 4: AVERAGE HOUSEHOLD CHARACTERISTICS AND THE PROBABILITY OF OWNING A HOME FOR HOUSEHOLDS WITH SELECTED CHARACTERISTICS, BY HOUSEHOLD AGE GROUP

	Symbolic representation	Household age group (age of husband)				
		25-34	35-44	45-54	55-64	65-74
Average household characteristics						
Assets (median)	$\bar{Z}$	16,318	46,367	63,818	70,000	68,909
Noninvestment income (median)	$\bar{Y}-\bar{Y}^*$	31,848	36,543	39,346	30,427	14,130
Persons aged 0-6 (mean)	$\bar{D}_1$	.841	.455	.055	.008	.000
Persons aged 7-12 (mean)	$\bar{D}_2$	.303	.827	.277	.030	.000
Persons aged 13-17 (mean)	$\bar{D}_3$	.042	.559	.566	.157	.015
Persons aged 18 and over (mean)	$\bar{D}_4$	1.988	2.153	2.821	2.542	2.207
Presence of children under 18 (mean)	$\bar{C}$	.694	.874	.550	.154	.016
Probability of owning a home						
Average household characteristics	as above	.7327	.9994	.9999	.9979	.9721
50% higher assets and income	$1.5\times(\bar{Z},\bar{Y}-\bar{Y}^*)$	.9396	1.0000	1.0000	1.0000	.9993
50% lower assets and income	$0.5\times(\bar{Z},\bar{Y}-\bar{Y}^*)$	.3785	.8793	.9366	.8372	.7487
50% larger household	$1.5\times(\bar{D}_1,\bar{D}_2,\bar{D}_3,\bar{D}_4-2)$	.7740	.9997	.9999	.9985	.9751
50% smaller household	$0.5\times(\bar{D}_1,\bar{D}_2,\bar{D}_3,\bar{D}_4-2)$	.6878	.9991	.9997	.9971	.9704
Two-adult household	$D_4=2; D_1=D_2=D_3=C=0$	.5139	.9946	.9988	.9951	.9673

NOTE: Variables not specified as altered in a probability calculation for a given age group are set at their mean values for that age group.

11. SUGGESTIONS FOR FUTURE RESEARCH AND DATA ACQUISITION

We have developed a theoretical foundation for the modelling of housing as consumption and investment, specified and estimated a system of econometric equations, and interpreted the results. The time available for the project was necessarily limited and we see a number of possibilities for extending our work but have not yet been able to pursue them. There are various avenues that can be explored in the future within the limitations of the data set with which we have been working, or by the use of other currently available data. We discuss these in the present section. In addition, we were specifically asked to examine the existing sets of household microdata and to suggest what additional data would be required for the estimation of a more comprehensive model than the one that we have been able to estimate so far. We do this also in the present section, giving attention both to relatively straightforward extensions that could be made by accessing data already available within Statistics Canada, and to more ambitious (and costly) extensions requiring new survey activity.

Under the heading of further research with the data set that has been available to us for this project we note several possibilities. One is an extension of the model to encompass "nonstandard" households -- households that lie outside the bounds represented by our restrictions, as set forth in section 7. We have deliberately avoided dealing with such households in constructing the present version of the model, on the grounds that we wanted to work with a relatively homogeneous set of units, and one that would conform as closely as possible to the assumptions underlying the theory. (To put it differently, if the model did

not fit this set of households it would be very unlikely to fit a more broadly defined set.) However, a reasonable extension would be to include some of the omitted categories -- single individuals living alone, households without both husband and wife present, households with husband's age less than 25, or 75 or older, households in smaller urban centres, and perhaps others. The model would probably require some ad hoc adjustments to its specifications to accommodate such categories.

Another extension would involve the incorporation of additional explanatory variables into the expenditure and tenure-choice equations, or rather experimental re-estimation of the model to see if it were improved by the inclusion of such variables. We did some preliminary experimentation with educational-category and regional dummy variables but were unable to follow this line of work to any satisfactory conclusion within the time available. In addition to education and geographic region, other variables on the 1982 FAMEX public use tape that might be of interest in this regard include occupation, mother tongue, immigrant-nonimmigrant status, and immigrant year of arrival.

A third type of extension with the present data set would involve exploration to see if formal account could be taken in the model of restrictions on borrowing by young households. Both the statistical evidence provided by the model and casual observation suggest that such restrictions may significantly constrain young households to levels of owned-housing consumption and investment below what they would choose if they had greater freedom to borrow against future earnings. Such constraints are difficult to impose

directly but we have in mind possibilities for inferring their effects by using parameter estimates for older households, on the assumption that for older households the constraints are less likely to be binding and the parameter estimates therefore are likely to be more indicative of freely determined optimum allocations of wealth and current expenditure.

Turning now to issues relating to the usefulness of available data, we note again what we have said elsewhere in the report, namely that the FAMEX surveys seem to be the only source of Canadian microdata with sufficient range of coverage to make possible econometric estimation of models involving both household expenditures on housing and nonhousing commodities and services, and housing viewed as investment. The other microdata sources do not include expenditure variables, and these variables are essential. In our opinion, the emphasis should therefore be on extracting as much useful information as possible from FAMEX and on exploring the possibilities for obtaining additional information by use of the FAMEX instrument in the future.

The most glaring deficiency of the 1982 FAMEX public use tape for estimating housing expenditure or demand models is the absence of price information. Closely related to this is the absence of geographic identification below the level of broad regions. Limited urban-rural and urban size group coding is provided but it is impossible to associate individual households with particular locations. If coding were provided to identify major urban centres (census metropolitan areas), households could be associated with the housing markets in those areas, and supplementary data from outside FAMEX could be brought to bear in the

analysis. Housing prices (both levels and variability), spatial price indexes for nonhousing goods, and other city-specific variables could be introduced into the model specification and estimation. We view the matter of geographic (urban) coding as one that should receive top priority.

The absence of detailed geographic coding on the FAMEX public use sample tape is, no doubt, a result of Statistics Canada's need to preserve confidentiality of individual households. We appreciate this need. However, it would seem possible to meet the confidentiality requirement and still have the benefits of geographic coding. One possibility is to contract with Statistics Canada to do all processing requiring individual records "in-house." (This has been done on other occasions in the past. It is currently being done in connection with a study being carried out by John Burbidge and Leslie Robb of McMaster University.) A second possibility is to add urban-area coding to the public use sample tape but filter the records on the tape so as to eliminate any means of identifying individual households. The expenditure categories could be collapsed into a smaller number, if that were helpful; households with very high incomes or housing values could be eliminated (or perhaps left on the tape but identified as units for which area coding could not be provided); and so on. If the reductions of information necessary to provide such coding on the public use tape were not too extensive, the second of the two alternatives would be preferable. However, both possibilities should be explored, in our opinion.

One variant of the "in-house" processing approach is to

have a model specified by researchers outside Statistics Canada and then estimated inside. Another variant (one that has been used in the past) is to specify the moment matrix to be calculated from the data, and have this calculated in Statistics Canada and then turned over to outside researchers. This is feasible if the model is linear in its parameters, and does not have to be estimated by an iterative technique. The advantage of the second approach is that it is more flexible and requires less back-and-forth communication between outside researchers and persons inside Statistics Canada who are doing the processing. (It is very hard to get things right the first time, or the first few times. It is also very hard to foresee final equation specifications at the outset of a project; much experimentation is generally required.) The second approach would be preferable, in our view, but again we think that both should be explored. In any event, we emphasize strongly the need for geographic identification that will allow households to be associated with particular local housing markets, and will allow supplementary information on local prices to be used in conjunction with the survey microdata.

Another class of information that is important for our type of model is wealth information. It would be highly desirable to have household assets and liabilities included in the FAMEX data set, along with the expenditure, income, demographic, and other information that is now included. The 1982 FAMEX microdata set includes the value of owned dwellings and the balance of mortgage principal owing, and this information has been extremely valuable to us. It allows the use of variables representing homeowners' housing equity and mortgage debt. However, other forms of tang-

ible assets are not covered by the survey and we have had to infer them from reported investment income. There is, of course, no way of measuring directly the present-value wealth represented by future noninvestment income streams, and this would have to be inferred or proxied in any event. However, the provision of nonhousing tangible asset and liability information would be a major improvement to the FAMEX set. (Parenthetically, we note that our suggestions should not be taken as criticism of Statistics Canada. We think that the agency should be congratulated for providing the combination of information that it now provides on the FAMEX tape. Our aim is merely to suggest ways of making the FAMEX data more useful in the future for housing studies.)

We introduced the idea of the disutility of a housing move in our initial theoretical specification of the household utility function, and the idea of real costs associated with moving in the original specification of the wealth constraint. However, in the model designed for econometric estimation we were forced to drop any notions relating to mover transaction costs since the data did not permit them to be taken account of. For the future, we suggest that the possibilities for obtaining survey information relating explicitly to movers and mover costs be explored. We have in mind the identification of movers, the types of move (rental to rental, rental to owning, etc.), the values of previous owned homes and new owned homes, the geographic distances associated with moves (same city, different city, etc.), and the specific costs, both of moving out of one home and of moving into another

(legal fees, real estate fees, costs of transporting household goods and members, etc.). Another (and especially important) type of information would be whether a move was job-related, and if so, whether the employer paid some or all of the costs. Some moving cost information has been collected in the past but it has not been provided on the 1982 public use tape, and in any event it appears not to be sufficiently comprehensive for the purposes that we have in mind. It should be noted that picking up mover households incidentally, as part of the general FAMEX survey, would probably yield a relatively small sample. It might be necessary to think in terms of a larger sample size than that of the usual survey, or perhaps some form of special-purpose sample restricted to mover households alone.

Longitudinal information on individual households would be extremely valuable for the modelling of housing consumption-investment decisions. It would allow household moves to be taken into account much more naturally and effectively than is the case with independent cross-section surveys. The acquisition of longitudinal data would no doubt require a major new survey initiative, and then the passage of time while the longitudinal data accumulated, unless of course historical records could be compiled with sufficient accuracy by respondent recall. The possibility of a longitudinal panel survey of some kind might well be considered if the budgetary situation seems to make it potentially feasible. A relatively much less expensive (but also less informative) procedure is what is sometimes called the "pseudo-panel" approach. It involves the use of consecutive sets of independent cross-section data to establish time-series relationships for individual popula-

tion cohorts. One can estimate separate econometric equations for each survey, and then use these to infer the behaviour of a given cohort as it ages and moves from one survey (and hence one set of equations) to another. Individuals cannot be tracked by this method, only the cohorts to which they belong. Nevertheless, the method has advantages and its use may be worth exploring. (This approach is being used in the Burbidge-Robb study mentioned above.) It would appear that there may be sufficient consistency in the variables reported in successive FAMEX surveys to make such an approach possible but the matter would require careful examination of the survey documentation to verify this.

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A P P E N D I X

ESTIMATED EQUATIONS

The thirteen equations that constitute the estimated model are presented in this appendix. The equations were estimated using SHAZAM 5.0 and the tabular material is reproduced directly from the SHAZAM printouts. Definitions of symbols representing variables in the printouts are also provided.

DEFINITIONS OF VARIABLES

The following symbols appear in the computer printouts of the estimated equations. We provide definitions and note the correspondence with symbols used in the main body of the report if the symbols are not the same.

AE1 -- product of husband's age group 1 dummy variable and total expenditure variable (same as $A_1 e_0$ for renters or $A_1 e_0^*$ for owners)

AE2,AE3,AE4,AE5 -- correspondingly defined for age groups 2,3,4,5

AGEB1 -- husband's age group 1 dummy variable, equal to 1 if husband is aged 25-34 and 0 otherwise (same as A_1)

AGEB2 -- correspondingly defined for age group 2, which is 35-44

AGEB3 -- correspondingly defined for age group 3, which is 45-54

AGEB4 -- correspondingly defined for age group 4, which is 55-64

AGEB5 -- correspondingly defined for age group 5, which is 65-74

AVMY1 -- product of husband's age group 1 dummy variable and total assets, the latter being calculated as housing value minus mortgage principal outstanding, plus imputed value of nonhousing assets (same as $A_1 Z$)

AVMY2,AVMY3,AVMY4,AVMY5 -- correspondingly defined for age groups 2,3,4,5

AYY1 -- product of husband's group 1 dummy variable and noninvestment income variable (same as $A_1 (Y-Y^*)$)

AYY2,AYY3,AYY4,AYY5 -- correspondingly defined for ages groups 2,3,4,5

C -- dummy variable for presence of children, equal to 1 if there are any children under the age of 18 and 0 otherwise

D1 -- number of person-years in demographic group 1, which is 0-6 years of age (e.g., one child aged 0-6 who was present in the household for the entire year would be represented by D1=1; two children 0-6 would be represented by D1=2.0)

D2 -- correspondingly defined for demographic group 2, which is 7-12 years of age

D3 -- correspondingly defined for demographic group 3, which is 13-17 years of age

D4 -- correspondingly defined for demographic group 4, which is 18 years of age and over

D1A1 -- product of husband's age group 1 dummy variable and number of person-years in household demographic group 1 (same as A_1D_1)

D1A2,...,D4A5 -- correspondingly defined for other husband's age groups and household demographic groups

TABLE A1: TOTAL EXPENDITURE EQUATION FOR RENTERS (DEPENDENT VARIABLE ϵ_0)

VARIABLE NAME	ESTIMATED COEFFICIENT	STANDARD ERROR	T-RATIO 944 DF	PARTIAL CORR.	STANDARDIZED COEFFICIENT	ELASTICITY AT MEANS
AVMY1	0.73634E-01	0.34772E-02	21.176	0.5675	0.38536E-01	0.36359E-02
AVMY2	0.87486E-01	0.44077E-02	19.849	0.5426	0.44751E-01	0.24774E-02
AVMY3	0.10427	0.47461E-02	21.971	0.5817	0.52855E-01	0.26597E-02
AVMY4	0.65550E-01	0.51323E-02	12.772	0.3839	0.47471E-01	0.27594E-02
AVMY5	0.68679E-01	0.68065E-02	10.090	0.3120	0.63259E-01	0.42883E-02
AYY1	0.42556	0.20097E-01	21.176	0.5675	0.76550	0.25626
AYY2	0.46868	0.23613E-01	19.849	0.5426	0.73825	0.13847
AYY3	0.57237	0.26052E-01	21.971	0.5817	0.80382	0.10667
AYY4	0.42070	0.32939E-01	12.772	0.3839	0.43091	0.52151E-01
AYY5	0.44426	0.44029E-01	10.090	0.3120	0.32100	0.33631E-01
AGEB1	9283.4	3619.1	2.5651	0.0832	0.51719	0.20084
AGEB2	3449.0	2176.7	1.5845	0.0515	0.15800	0.34185E-01
AGEB3	4699.6	1806.3	2.6018	0.0844	0.17083	0.26202E-01
AGEB4	2142.7	1958.7	1.0940	0.0356	0.74301E-01	0.10721E-01
AGEB5	8291.1	2664.4	3.1118	0.1008	0.26440	0.34372E-01
D1A1	90.502	326.15	0.27749	0.0090	0.60708E-02	0.11854E-02
D1A2	150.17	532.21	0.28216	0.0092	0.62863E-02	0.65792E-03
D1A3	2043.2	1741.5	1.1733	0.0382	0.23076E-01	0.77890E-03
D1A4	-4244.7	5228.9	-0.81177	-0.0264	-0.15202E-01	-0.20227E-03
D2A1	909.79	495.00	1.8380	0.0597	0.35958E-01	0.39017E-02
D2A2	2.6074	448.26	0.58167E-02	0.0002	0.13250E-03	0.15903E-04
D2A3	1024.2	819.97	1.2491	0.0406	0.26523E-01	0.15373E-02
D2A4	-1335.5	2253.3	-0.59267	-0.0193	-0.11706E-01	-0.25455E-03
D3A1	1981.0	1077.9	1.8378	0.0597	0.34629E-01	0.13216E-02
D3A2	837.65	468.27	1.7888	0.0581	0.39900E-01	0.42065E-02
D3A3	885.36	618.14	1.4323	0.0466	0.31978E-01	0.25102E-02
D3A4	2221.6	1601.7	1.3870	0.0451	0.28565E-01	0.11645E-02
D4A1	-50.319	1859.8	-0.27057E-01	-0.0009	-0.55492E-02	-0.21457E-02
D4A2	2536.1	974.24	2.6031	0.0844	0.25089	0.53147E-01
D4A3	113.66	621.02	0.18302	0.0060	0.10973E-01	0.15913E-02
D4A4	1989.2	831.63	2.3919	0.0776	0.17001	0.23575E-01
D4A5	-252.33	1260.9	-0.20012	-0.0065	-0.18243E-01	-0.23146E-02

```
!_RESTRICT AYY1 - (.36621/.063364)*AVMY1 = 0.
!_RESTRICT AYY2 - (.38045/.071017)*AVMY2 = 0.
!_RESTRICT AYY3 - (.38710/.070521)*AVMY3 = 0.
!_RESTRICT AYY4 - (.39609/.061715)*AVMY4 = 0.
!_RESTRICT AYY5 - (.30578/.047271)*AVMY5 = 0.
```

```
 R-SQUARE =  0.6756      R-SQUARE ADJUSTED =  0.6667
VARIANCE OF THE ESTIMATE =  0.26761E+08
STANDARD ERROR OF THE ESTIMATE =   5173.1
LOG OF THE LIKELIHOOD FUNCTION = -9667.34
```

ANALYSIS OF VARIANCE - FROM MEAN

	SS	DF	MS
REGRESSION	0.52619E+11	26.	0.20238E+10
ERROR	0.25263E+11	944.	0.26761E+08
TOTAL	0.77882E+11	970.	0.80291E+08

ANALYSIS OF VARIANCE - FROM ZERO

	SS	DF	MS
REGRESSION	0.50617E+12	27.	0.18747E+11
ERROR	0.25263E+11	944.	0.26761E+08
TOTAL	0.53144E+12	971.	0.54731E+09

TABLE A2: SHELTER EXPENDITURE EQUATION FOR RENTERS (DEPENDENT VARIABLE e_{h0})

VARIABLE NAME	ESTIMATED COEFFICIENT	STANDARD ERROR	T-RATIO 949 DF	PARTIAL CORR.	STANDARDIZED COEFFICIENT	ELASTICITY AT MEANS
AE1	0.11160	0.10116E-01	11.033	0.3372	0.69806	0.23829
AE2	0.12650	0.11823E-01	10.699	0.3281	0.70808	0.13543
AE3	0.15545	0.11359E-01	13.686	0.4060	0.75875	0.10133
AE4	0.16304	0.19028E-01	8.5683	0.2680	0.53906	0.67312E-01
AE5	0.16043	0.21508E-01	7.4592	0.2353	0.45573	0.51551E-01
D1A1	176.14	98.105	1.7954	0.0582	0.55934E-01	0.10594E-01
D1A2	394.83	153.39	2.5740	0.0833	0.78244E-01	0.79430E-02
D1A3	1593.2	522.58	3.0488	0.0985	0.85181E-01	0.27888E-02
D1A4	2136.3	1575.4	1.3560	0.0440	0.36220E-01	0.46744E-03
D2A1	328.34	149.74	2.1927	0.0710	0.61433E-01	0.64659E-02
D2A2	313.59	130.53	2.4023	0.0777	0.75439E-01	0.87827E-02
D2A3	337.95	241.32	1.4004	0.0454	0.41429E-01	0.23293E-02
D2A4	1298.3	682.12	1.9033	0.0617	0.53872E-01	0.11363E-02
D3A1	-46.278	327.40	-0.14135	-0.0046	-0.38296E-02	-0.14176E-03
D3A2	-33.187	140.88	-0.23556	-0.0076	-0.74833E-02	-0.76524E-03
D3A3	199.73	187.26	1.0666	0.0346	0.34151E-01	0.26003E-02
D3A4	-207.58	484.37	-0.42855	-0.0139	-0.12635E-01	-0.49960E-03
D4A1	1046.8	123.37	8.4849	0.2655	0.54650	0.20497
D4A2	788.65	155.39	5.0752	0.1626	0.36934	0.75890E-01
D4A3	275.86	120.00	2.2988	0.0744	0.12607	0.17734E-01
D4A4	561.00	154.92	3.6213	0.1167	0.22697	0.30529E-01
D4A5	554.54	177.32	3.1273	0.1010	0.18979	0.23357E-01

R-SQUARE = 0.3279 R-SQUARE ADJUSTED = 0.3131
VARIANCE OF THE ESTIMATE = 0.24611E+07
STANDARD ERROR OF THE ESTIMATE = 1568.8
LOG OF THE LIKELIHOOD FUNCTION = -8511.34
RAW MOMENT R-SQUARE = 0.9065

ANALYSIS OF VARIANCE - FROM ZERO

	SS	DF	MS	F
REGRESSION	0.22651E+11	22.	0.10296E+10	418.343
ERROR	0.23356E+10	949.	0.24611E+07	
TOTAL	0.24987E+11	971.	0.25733E+08	

TABLE A3: FOOD EXPENDITURE EQUATION FOR RENTERS (DEPENDENT VARIABLE ϵ_{i0}, i=1)

VARIABLE NAME	ESTIMATED COEFFICIENT	STANDARD ERROR	T-RATIO 949 DF	PARTIAL CORR.	STANDARDIZED COEFFICIENT	ELASTICITY AT MEANS
AE1	0.11475	0.84134E-02	13.639	0.4048	0.73067	0.25839
AE2	0.12342	0.98335E-02	12.551	0.3773	0.70327	0.13935
AE3	0.97276E-01	0.94474E-02	10.297	0.3170	0.48335	0.66870E-01
AE4	0.12749	0.15826E-01	8.0557	0.2530	0.42913	0.55512E-01
AE5	0.98188E-01	0.17889E-01	5.4888	0.1754	0.28394	0.33274E-01
D1A1	391.54	81.596	4.7985	0.1539	0.12657	0.24836E-01
D1A2	69.902	127.58	0.54790	0.0178	0.14102E-01	0.14831E-02
D1A3	-155.97	434.64	-0.35886	-0.0116	-0.84894E-02	-0.28794E-03
D1A4	568.17	1310.3	0.43361	0.0141	0.98065E-02	0.13111E-03
D2A1	407.38	124.54	3.2710	0.1056	0.77594E-01	0.84606E-02
D2A2	447.76	108.57	4.1242	0.1327	0.10966	0.13226E-01
D2A3	423.99	200.71	2.1124	0.0684	0.52914E-01	0.30820E-02
D2A4	551.49	567.33	0.97208	0.0315	0.23296E-01	0.50905E-03
D3A1	584.46	272.31	2.1463	0.0695	0.49236E-01	0.18882E-02
D3A2	428.40	117.17	3.6560	0.1179	0.98341E-01	0.10418E-01
D3A3	642.90	155.75	4.1278	0.1328	0.11191	0.88272E-02
D3A4	433.81	402.86	1.0768	0.0349	0.26881E-01	0.11012E-02
D4A1	690.93	102.61	6.7334	0.2135	0.36721	0.14268
D4A2	833.84	129.24	6.4517	0.2050	0.39754	0.84622E-01
D4A3	1024.5	99.806	10.265	0.3161	0.47662	0.69458E-01
D4A4	656.49	128.85	5.0951	0.1632	0.27039	0.37678E-01
D4A5	858.66	147.48	5.8222	0.1857	0.29917	0.38143E-01

R-SQUARE = 0.5182 R-SQUARE ADJUSTED = 0.5075
VARIANCE OF THE ESTIMATE = 0.17025E+07
STANDARD ERROR OF THE ESTIMATE = 1304.8
LOG OF THE LIKELIHOOD FUNCTION = -8332.42
RAW MOMENT R-SQUARE = 0.9288

ANALYSIS OF VARIANCE - FROM ZERO

	SS	DF	MS	F
REGRESSION	0.21078E+11	22.	0.95808E+09	562.753
ERROR	0.16157E+10	949.	0.17025E+07	
TOTAL	0.22693E+11	971.	0.23371E+08	

TABLE A4: CLOTHING EXPENDITURE EQUATION FOR RENTERS (DEPENDENT VARIABLE e_{i0}, i=2)

VARIABLE NAME	ESTIMATED COEFFICIENT	STANDARD ERROR	T-RATIO 949 DF	PARTIAL CORR.	STANDARDIZED COEFFICIENT	ELASTICITY AT MEANS
AE1	0.11480	0.63791E-02	17.997	0.5044	0.98653	0.65460
AE2	0.96758E-01	0.74558E-02	12.978	0.3882	0.74406	0.27664
AE3	0.96788E-01	0.71631E-02	13.512	0.4017	0.64901	0.16847
AE4	0.97563E-01	0.11999E-01	8.1308	0.2552	0.44318	0.10757
AE5	0.69405E-01	0.13563E-01	5.1171	0.1639	0.27085	0.59555E-01
D1A1	-187.17	61.866	-3.0254	-0.0977	-0.81656E-01	-0.30064E-01
D1A2	-118.76	96.732	-1.2278	-0.0398	-0.32334E-01	-0.63804E-02
D1A3	55.381	329.54	0.16805	0.0055	0.40678E-02	0.25888E-03
D1A4	-509.45	993.49	-0.51279	-0.0166	-0.11866E-01	-0.29768E-03
D2A1	35.827	94.429	0.37941	0.0123	0.92092E-02	0.18841E-02
D2A2	-4.9824	82.317	-0.60527E-01	-0.0020	-0.16467E-02	-0.37265E-03
D2A3	-52.806	152.18	-0.34699	-0.0113	-0.88934E-02	-0.97195E-03
D2A4	-315.59	430.15	-0.73368	-0.0238	-0.17990E-01	-0.73763E-03
D3A1	-48.613	206.46	-0.23546	-0.0076	-0.55266E-02	-0.39768E-03
D3A2	-1.1556	88.842	-0.13008E-01	-0.0004	-0.35800E-03	-0.71161E-04
D3A3	157.76	118.09	1.3360	0.0433	0.37059E-01	0.54849E-02
D3A4	82.162	305.45	0.26899	0.0087	0.68705E-02	0.52810E-03
D4A1	-262.00	77.801	-3.3676	-0.1087	-0.18791	-0.13700
D4A2	-137.38	97.993	-1.4020	-0.0455	-0.88390E-01	-0.35304E-01
D4A3	-125.76	75.673	-1.6618	-0.0539	-0.78956E-01	-0.21590E-01
D4A4	-192.09	97.692	-1.9663	-0.0637	-0.10677	-0.27916E-01
D4A5	-32.131	111.82	-0.28734	-0.0093	-0.15108E-01	-0.36141E-02

R-SQUARE = 0.4956 R-SQUARE ADJUSTED = 0.4844
VARIANCE OF THE ESTIMATE = 0.97872E+06
STANDARD ERROR OF THE ESTIMATE = 989.30
LOG OF THE LIKELIHOOD FUNCTION = -8063.65
RAW MOMENT R-SQUARE = 0.8088

ANALYSIS OF VARIANCE - FROM ZERO

	SS	DF	MS	F
REGRESSION	0.39289E+10	22.	0.17859E+09	182.469
ERROR	0.92880E+09	949.	0.97872E+06	
TOTAL	0.48577E+10	971.	0.50028E+07	

TABLE A5: OTHER NONSHELTER EXPENDITURE EQUATION FOR RENTERS (DEPENDENT VARIABLE e_{i0}, i=3)

VARIABLE NAME	ESTIMATED COEFFICIENT	STANDARD ERROR	T-RATIO 949 DF	PARTIAL CORR.	STANDARDIZED COEFFICIENT	ELASTICITY AT MEANS
AE1	0.65885	0.13086E-01	50.347	0.8530	1.2702	0.61995
AE2	0.65333	0.15295E-01	42.716	0.8111	1.1272	0.30826
AE3	0.65048	0.14694E-01	44.268	0.8208	0.97858	0.18685
AE4	0.61191	0.24615E-01	24.859	0.6280	0.62360	0.11134
AE5	0.67197	0.27824E-01	24.151	0.6170	0.58833	0.95155E-01
D1A1	-380.50	126.91	-2.9982	-0.0969	-0.37242E-01	-0.10086E-01
D1A2	-345.97	198.44	-1.7435	-0.0565	-0.21132E-01	-0.30673E-02
D1A3	-1492.6	676.03	-2.2079	-0.0715	-0.24597E-01	-0.11514E-02
D1A4	-2195.0	2038.0	-1.0770	-0.0349	-0.11471E-01	-0.21166E-03
D2A1	-771.55	193.71	-3.9830	-0.1282	-0.44493E-01	-0.66958E-02
D2A2	-756.36	168.86	-4.4791	-0.1439	-0.56083E-01	-0.93355E-02
D2A3	-709.14	312.18	-2.2715	-0.0735	-0.26795E-01	-0.21540E-02
D2A4	-1534.2	882.41	-1.7387	-0.0563	-0.19621E-01	-0.59176E-03
D3A1	-489.57	423.54	-1.1559	-0.0375	-0.12487E-01	-0.66090E-03
D3A2	-394.05	182.25	-2.1621	-0.0700	-0.27387E-01	-0.40043E-02
D3A3	-1000.4	242.25	-4.1296	-0.1329	-0.52722E-01	-0.57397E-02
D3A4	-308.39	626.59	-0.49218	-0.0160	-0.57856E-02	-0.32711E-03
D4A1	-1475.8	159.60	-9.2465	-0.2875	-0.23746	-0.12734
D4A2	-1485.1	201.02	-7.3877	-0.2332	-0.21437	-0.62979E-01
D4A3	-1174.6	155.24	-7.5663	-0.2385	-0.16545	-0.33276E-01
D4A4	-1025.4	200.40	-5.1166	-0.1638	-0.12787	-0.24592E-01
D4A5	-1381.1	229.39	-6.0206	-0.1918	-0.14569	-0.25636E-01

R-SQUARE = 0.8932 R-SQUARE ADJUSTED = 0.8908
VARIANCE OF THE ESTIMATE = 0.41187E+07
STANDARD ERROR OF THE ESTIMATE = 2029.4
LOG OF THE LIKELIHOOD FUNCTION = -8761.33
RAW MOMENT R-SQUARE = 0.9735

ANALYSIS OF VARIANCE - FROM ZERO

	SS	DF	MS	F
REGRESSION	0.14343E+12	22.	0.65197E+10	1582.974
ERROR	0.39086E+10	949.	0.41187E+07	
TOTAL	0.14734E+12	971.	0.15174E+09	

TABLE A6: TOTAL NONSHELTER EXPENDITURE EQUATION FOR RENTERS (DEPENDENT VARIABLE Σe_{i0})

VARIABLE NAME	ESTIMATED COEFFICIENT	STANDARD ERROR	T-RATIO 949 DF	PARTIAL CORR.	STANDARDIZED COEFFICIENT	ELASTICITY AT MEANS
AE1	0.88840	0.10116E-01	87.824	0.9436	1.3116	0.52811
AE2	0.87350	0.11823E-01	73.881	0.9230	1.1541	0.26037
AE3	0.84455	0.11359E-01	74.351	0.9238	0.97299	0.15326
AE4	0.83696	0.19028E-01	43.986	0.8191	0.65321	0.96207E-01
AE5	0.83957	0.21508E-01	39.035	0.7850	0.56293	0.75107E-01
D1A1	-176.14	98.105	-1.7954	-0.0582	-0.13203E-01	-0.29496E-02
D1A2	-394.83	153.39	-2.5740	-0.0833	-0.18469E-01	-0.22114E-02
D1A3	-1593.2	522.58	-3.0488	-0.0985	-0.20106E-01	-0.77645E-03
D1A4	-2136.3	1575.4	-1.3560	-0.0440	-0.85494E-02	-0.13014E-03
D2A1	-328.34	149.74	-2.192/	-0.0710	-0.14501E-01	-0.18002E-02
D2A2	-313.59	130.53	-2.4023	-0.0777	-0.17807E-01	-0.24452E-02
D2A3	-337.95	241.32	-1.4004	-0.0454	-0.97790E-02	-0.64850E-03
D2A4	-1298.3	682.12	-1.9033	-0.0617	-0.12716E-01	-0.31636E-03
D3A1	46.278	327.40	0.14135	0.0046	0.90394E-03	0.39469E-04
D3A2	33.187	140.88	0.23556	0.0076	0.17664E-02	0.21305E-03
D3A3	-199.73	187.26	-1.0666	-0.0346	-0.80611E-02	-0.72396E-03
D3A4	207.58	484.37	0.42855	0.0139	0.29823E-02	0.13910E-03
D4A1	-1046.8	123.37	-8.4849	-0.2655	-0.12900	-0.57066E-01
D4A2	-788.65	155.39	-5.0752	-0.1626	-0.87179E-01	-0.21129E-01
D4A3	-275.86	120.00	-2.2988	-0.0744	-0.29757E-01	-0.49373E-02
D4A4	-561.00	154.92	-3.6213	-0.1167	-0.53575E-01	-0.84997E-02
D4A5	-554.54	177.32	-3.1273	-0.1010	-0.44799E-01	-0.65029E-02

R-SQUARE = 0.9626 R-SQUARE ADJUSTED = 0.9617
VARIANCE OF THE ESTIMATE = 0.24611E+07
STANDARD ERROR OF THE ESTIMATE = 1568.8
LOG OF THE LIKELIHOOD FUNCTION = -8511.34
RAW MOMENT R-SQUARE = 0.9931

ANALYSIS OF VARIANCE - FROM ZERO

	SS	DF	MS	F
REGRESSION	0.33755E+12	22.	0.15343E+11	6234.320
ERROR	0.23356E+10	949.	0.24611E+07	
TOTAL	0.33989E+12	971.	0.35004E+09	

TABLE A7: TOTAL EXPENDITURE EQUATION FOR OWNERS (DEPENDENT VARIABLE ϵ_0^*)

VARIABLE NAME	ESTIMATED COEFFICIENT	STANDARD ERROR	T-RATIO 3450 DF	PARTIAL CORR.	STANDARDIZED COEFFICIENT	ELASTICITY AT MEANS
AVMY1	0.63364E-01	0.59935E-02	10.572	0.1771	0.13976	0.21043E-01
AVMY2	0.71017E-01	0.37198E-02	19.092	0.3091	0.26235	0.49337E-01
AVMY3	0.70521E-01	0.36249E-02	19.455	0.3144	0.29232	0.50342E-01
AVMY4	0.61715E-01	0.33732E-02	18.296	0.2974	0.26926	0.40706E-01
AVMY5	0.47271E-01	0.37084E-02	12.747	0.2121	0.18055	0.18992E-01
AYY1	0.36621	0.17014E-01	21.524	0.3441	0.52773	0.10709
AYY2	0.38045	0.13162E-01	28.906	0.4416	0.65981	0.15942
AYY3	0.38710	0.13527E-01	28.617	0.4380	0.66046	0.13566
AYY4	0.39609	0.15314E-01	25.864	0.4030	0.53823	0.93152E-01
AYY5	0.30578	0.27454E-01	11.138	0.1863	0.18878	0.21686E-01
AGEB1	-3161.5	16500.	-0.19161	-0.0033	-0.11040	-0.24558E-01
AGEB2	2406.0	1258.9	1.9112	0.0325	0.92381E-01	0.24750E-01
AGEB3	781.08	894.75	0.87296	0.0149	0.27775E-01	0.63801E-02
AGEB4	561.06	895.03	0.62686	0.0107	0.18509E-01	0.37491E-02
AGEB5	772.81	1565.4	0.49369	0.0084	0.20139E-01	0.29404E-02
D1A1	415.52	299.58	1.3870	0.0236	0.19934E-01	0.31842E-02
D1A2	-154.40	328.33	-0.47026	-0.0080	-0.56821E-02	-0.72620E-03
D1A3	310.27	992.43	0.31264	0.0053	0.31947E-02	0.13408E-03
D1A4	3348.7	1991.1	1.6818	0.0286	0.16084E-01	0.17647E-03
D2A1	988.62	379.97	2.6018	0.0443	0.28545E-01	0.28282E-02
D2A2	575.14	274.78	2.0931	0.0356	0.28851E-01	0.51586E-02
D2A3	442.79	442.75	1.0001	0.0170	0.11010E-01	0.10080E-02
D2A4	-2511.1	1600.1	-1.5693	-0.0267	-0.15394E-01	-0.47639E-03
D3A1	1666.8	934.47	1.7836	0.0304	0.17686E-01	0.63513E-03
D3A2	1402.2	290.60	4.8252	0.0819	0.58240E-01	0.82285E-02
D3A3	1487.5	333.02	4.4666	0.0758	0.52631E-01	0.69783E-02
D3A4	1162.3	605.91	1.9182	0.0326	0.19624E-01	0.12862E-02
D3A5	4622.9	2662.3	1.7364	0.0295	0.16726E-01	0.31670E-03
D4A1	5304.4	8243.4	0.64347	0.0110	0.37026	0.82350E-01
D4A2	2382.7	456.63	5.2180	0.0885	0.20466	0.52971E-01
D4A3	2400.8	260.12	9.2297	0.1552	0.26168	0.56242E-01
D4A4	2355.3	335.04	7.0300	0.1188	0.21325	0.40463E-01
D4A5	3404.1	708.49	4.8047	0.0815	0.20171	0.28562E-01

R-SQUARE = 0.6857 R-SQUARE ADJUSTED = 0.6828
VARIANCE OF THE ESTIMATE = 0.43414E+08
STANDARD ERROR OF THE ESTIMATE = 6589.0
LOG OF THE LIKELIHOOD FUNCTION = -35552.1

ANALYSIS OF VARIANCE - FROM MEAN

	SS	DF	MS	F
REGRESSION	0.32672E+12	32.	0.10210E+11	235.177
ERROR	0.14978E+12	3450.	0.43414E+08	
TOTAL	0.47650E+12	3482.	0.13685E+09	

ANALYSIS OF VARIANCE - FROM ZERO

	SS	DF	MS	F
REGRESSION	0.29113E+13	33.	0.88221E+11	2032.065
ERROR	0.14978E+12	3450.	0.43414E+08	
TOTAL	0.30611E+13	3483.	0.87886E+09	

TABLE A8: SHELTER EXPENDITURE EQUATION FOR OWNERS (DEPENDENT VARIABLE ϵ^{*}_{h0})

VARIABLE NAME	ESTIMATED COEFFICIENT	STANDARD ERROR	T-RATIO 3460 DF	PARTIAL CORR.	STANDARDIZED COEFFICIENT	ELASTICITY AT MEANS
AE1	0.20314	0.12638E-01	16.074	0.2636	0.61107	0.16934
AE2	0.20431	0.81278E-02	25.137	0.3930	0.80941	0.26456
AE3	0.24930	0.81392E-02	30.630	0.4619	0.99181	0.27707
AE4	0.24318	0.94382E-02	25.766	0.4012	0.78796	0.18849
AE5	0.27503	0.17464E-01	15.749	0.2586	0.49653	0.86311E-01
D1A1	96.123	128.80	0.74632	0.0127	0.14938E-01	0.31886E-02
D1A2	604.21	131.02	4.6115	0.0782	0.72028E-01	0.12302E-01
D1A3	56.271	435.27	0.12928	0.0022	0.18768E-02	0.10526E-03
D1A4	-487.11	877.69	-0.55499	-0.0094	-0.75788E-02	-0.11112E-03
D2A1	-471.40	166.43	-2.8325	-0.0481	-0.44091E-01	-0.58375E-02
D2A2	271.84	108.46	2.5064	0.0426	0.44173E-01	0.10555E-01
D2A3	350.21	191.39	1.8298	0.0311	0.28208E-01	0.34512E-02
D2A4	-500.91	702.05	-0.71349	-0.0121	-0.99469E-02	-0.41136E-03
D3A1	-505.11	411.03	-1.2289	-0.0209	-0.17361E-01	-0.83316E-03
D3A2	-168.35	126.60	-1.3298	-0.0226	-0.22651E-01	-0.42765E-02
D3A3	-33.403	146.12	-0.22860	-0.0039	-0.38285E-02	-0.67833E-03
D3A4	-146.95	266.23	-0.55197	-0.0094	-0.80369E-02	-0.70396E-03
D3A5	877.88	1173.7	0.74796	0.0127	0.10289E-01	0.26034E-03
D4A1	328.99	187.24	1.7571	0.0299	0.74386E-01	0.22109E-01
D4A2	34.429	117.58	0.29280	0.0050	0.95793E-02	0.33132E-02
D4A3	-474.66	89.400	-5.3094	-0.0899	-0.16759	-0.48133E-01
D4A4	-140.35	103.64	-1.3542	-0.0230	-0.41162E-01	-0.10437E-01
D4A5	139.77	162.54	0.85990	0.0146	0.26827E-01	0.50763E-02

R-SQUARE = 0.3582 R-SQUARE ADJUSTED = 0.3541
VARIANCE OF THE ESTIMATE = 0.84237E+07
STANDARD ERROR OF THE ESTIMATE = 2902.4
LOG OF THE LIKELIHOOD FUNCTION = -32701.6
RAW MOMENT R-SQUARE = 0.8410

ANALYSIS OF VARIANCE - FROM ZERO

	SS	DF	MS	F
REGRESSION	0.15420E+12	23.	0.67043E+10	795.885
ERROR	0.29146E+11	3460.	0.84237E+07	
TOTAL	0.18334E+12	3483.	0.52640E+08	

TABLE A9: FOOD EXPENDITURE EQUATION FOR OWNERS (DEPENDENT VARIABLE e_{i0}, i=1)

VARIABLE NAME	ESTIMATED COEFFICIENT	STANDARD ERROR	T-RATIO 3460 DF	PARTIAL CORR.	STANDARDIZED COEFFICIENT	ELASTICITY AT MEANS
AE1	0.10089	0.64471E-02	15.649	0.2571	0.51262	0.10055
AE2	0.87639E-01	0.41463E-02	21.137	0.3382	0.58645	0.13567
AE3	0.95645E-01	0.41521E-02	23.035	0.3646	0.64271	0.12708
AE4	0.80766E-01	0.48148E-02	16.775	0.2742	0.44203	0.74843E-01
AE5	0.87052E-01	0.89090E-02	9.7712	0.1639	0.26546	0.32661E-01
D1A1	278.29	65.703	4.2356	0.0718	0.73049E-01	0.11037E-01
D1A2	266.08	66.840	3.9807	0.0675	0.53576E-01	0.64766E-02
D1A3	575.94	222.05	2.5938	0.0441	0.32447E-01	0.12880E-02
D1A4	-363.74	447.74	-0.81240	-0.0138	-0.95593E-02	-0.99204E-04
D2A1	664.61	84.901	7.8280	0.1319	0.10500	0.98395E-02
D2A2	501.96	55.329	9.0722	0.1524	0.13777	0.23300E-01
D2A3	497.63	97.635	5.0968	0.0863	0.67701E-01	0.58630E-02
D2A4	669.57	358.14	1.8696	0.0318	0.22458E-01	0.65740E-03
D3A1	538.31	209.68	2.5673	0.0436	0.31253E-01	0.10616E-02
D3A2	684.01	64.584	10.591	0.1772	0.15544	0.20773E-01
D3A3	748.85	74.540	10.046	0.1684	0.14498	0.18181E-01
D3A4	929.55	135.81	6.8443	0.1156	0.85872E-01	0.53238E-02
D3A5	918.06	598.75	1.5333	0.0261	0.18175E-01	0.32550E-03
D4A1	868.25	95.517	9.0900	0.1527	0.33160	0.69760E-01
D4A2	1059.4	59.984	17.661	0.2876	0.49787	0.12188
D4A3	846.06	45.606	18.551	0.3008	0.50456	0.10257
D4A4	1015.1	52.872	19.198	0.3103	0.50284	0.90247E-01
D4A5	829.29	82.916	10.002	0.1676	0.26886	0.36010E-01

R-SQUARE = 0.5235 R-SQUARE ADJUSTED = 0.5204
VARIANCE OF THE ESTIMATE = 0.21922E+07
STANDARD ERROR OF THE ESTIMATE = 1480.6
LOG OF THE LIKELIHOOD FUNCTION = -30357.2
RAW MOMENT R-SQUARE = 0.9325

ANALYSIS OF VARIANCE - FROM ZERO

	SS	DF	MS	F
REGRESSION	0.10483E+12	23.	0.45578E+10	2079.132
ERROR	0.75850E+10	3460.	0.21922E+07	
TOTAL	0.11242E+12	3483.	0.32275E+08	

TABLE A10: CLOTHING EXPENDITURE EQUATION FOR OWNERS (DEPENDENT VARIABLE e_{i0}, i=2)

VARIABLE NAME	ESTIMATED COEFFICIENT	STANDARD ERROR	T-RATIO 3460 DF	PARTIAL CORR.	STANDARDIZED COEFFICIENT	ELASTICITY AT MEANS
AE1	0.93479E-01	0.59014E-02	15.840	0.2600	0.55120	0.21538
AE2	0.98523E-01	0.37954E-02	25.959	0.4037	0.76511	0.35262
AE3	0.84207E-01	0.38007E-02	22.156	0.3525	0.65669	0.25867
AE4	0.86916E-01	0.44073E-02	19.721	0.3179	0.55205	0.18621
AE5	0.60345E-01	0.81550E-02	7.3998	0.1248	0.21356	0.52344E-01
D1A1	-148.39	60.142	-2.4674	-0.0419	-0.45204E-01	-0.13606E-01
D1A2	-262.52	61.183	-4.2908	-0.0728	-0.61346E-01	-0.14773E-01
D1A3	-228.49	203.25	-1.1242	-0.0191	-0.14939E-01	-0.11814E-02
D1A4	-753.01	409.85	-1.8373	-0.0312	-0.22966E-01	-0.47480E-03
D2A1	40.590	77.715	0.52229	0.0089	0.74419E-02	0.13893E-02
D2A2	-82.559	50.646	-1.6301	-0.0277	-0.26298E-01	-0.88599E-02
D2A3	48.169	89.372	0.53897	0.0092	0.76052E-02	0.13121E-02
D2A4	-380.51	327.83	-1.1607	-0.0197	-0.14812E-01	-0.86373E-03
D3A1	202.12	191.94	1.0531	0.0179	0.13618E-01	0.92153E-03
D3A2	195.60	59.118	3.3086	0.0562	0.51585E-01	0.13733E-01
D3A3	216.53	68.231	3.1735	0.0539	0.48650E-01	0.12154E-01
D3A4	262.79	124.32	2.1138	0.0359	0.28173E-01	0.34796E-02
D3A5	-4.3997	548.07	-0.80276E-02	-0.0001	-0.10108E-03	-0.36064E-05
D4A1	-96.282	87.432	-1.1012	-0.0187	-0.42674E-01	-0.17884E-01
D4A2	-54.946	54.907	-1.0007	-0.0170	-0.29968E-01	-0.14615E-01
D4A3	56.940	41.746	1.3639	0.0232	0.39408E-01	0.15960E-01
D4A4	-90.444	48.397	-1.8688	-0.0318	-0.51996E-01	-0.18591E-01
D4A5	-24.146	75.898	-0.31814	-0.0054	-0.90851E-02	-0.24240E-02

R-SQUARE = 0.4622 R-SQUARE ADJUSTED = 0.4588
VARIANCE OF THE ESTIMATE = 0.18368E+07
STANDARD ERROR OF THE ESTIMATE = 1355.3
LOG OF THE LIKELIHOOD FUNCTION = -30049.2
RAW MOMENT R-SQUARE = 0.7872

ANALYSIS OF VARIANCE - FROM ZERO

	SS	DF	MS	F
REGRESSION	0.23517E+11	23.	0.10225E+10	556.659
ERROR	0.63553E+10	3460.	0.18368E+07	
TOTAL	0.29872E+11	3483.	0.85766E+07	

TABLE A11: OTHER NONSHELTER EXPENDITURE EQUATION FOR OWNERS (DEPENDENT VARIABLE e_{i0}, i=3)

VARIABLE NAME	ESTIMATED COEFFICIENT	STANDARD ERROR	T-RATIO 3460 DF	PARTIAL CORR.	STANDARDIZED COEFFICIENT	ELASTICITY AT MEANS
AE1	0.60249	0.13469E-01	44.731	0.6053	0.85379	0.23573
AE2	0.60953	0.86624E-02	70.364	0.7672	1.1376	0.37045
AE3	0.57084	0.86746E-02	65.806	0.7456	1.0699	0.29777
AE4	0.58913	0.10059E-01	58.567	0.7056	0.89928	0.21433
AE5	0.57757	0.18613E-01	31.031	0.4666	0.49122	0.85075E-01
D1A1	-226.02	137.27	-1.6466	-0.0280	-0.16547E-01	-0.35191E-02
D1A2	-607.77	139.64	-4.3523	-0.0738	-0.34131E-01	-0.58079E-02
D1A3	-403.72	463.90	-0.87028	-0.0148	-0.63435E-02	-0.35446E-03
D1A4	1603.9	935.43	1.7146	0.0291	0.11756E-01	0.17173E-03
D2A1	-233.79	177.38	-1.3181	-0.0224	-0.10301E-01	-0.13589E-02
D2A2	-691.24	115.59	-5.9800	-0.1011	-0.52915E-01	-0.12597E-01
D2A3	-896.01	203.98	-4.3926	-0.0745	-0.33998E-01	-0.41445E-02
D2A4	211.85	748.23	0.28314	0.0048	0.19819E-02	0.81661E-04
D3A1	-235.33	438.07	-0.53720	-0.0091	-0.38105E-02	-0.18220E-03
D3A2	-711.25	134.93	-5.2713	-0.0893	-0.45080E-01	-0.84801E-02
D3A3	-931.98	155.73	-5.9846	-0.1012	-0.50322E-01	-0.88835E-02
D3A4	-1045.4	283.74	-3.6843	-0.0625	-0.26934E-01	-0.23506E-02
D3A5	-1791.5	1250.9	-1.4322	-0.0243	-0.98917E-02	-0.24937E-03
D4A1	-1101.0	199.55	-5.5171	-0.0934	-0.11727	-0.34727E-01
D4A2	-1038.8	125.32	-8.2896	-0.1395	-0.13617	-0.46924E-01
D4A3	-428.34	95.281	-4.4955	-0.0762	-0.71244E-01	-0.20387E-01
D4A4	-784.27	110.46	-7.1000	-0.1198	-0.10836	-0.27375E-01
D4A5	-944.91	173.23	-5.4547	-0.0923	-0.85441E-01	-0.16108E-01

R-SQUARE = 0.8382 R-SQUARE ADJUSTED = 0.8372
VARIANCE OF THE ESTIMATE = 0.95684E+07
STANDARD ERROR OF THE ESTIMATE = 3093.3
LOG OF THE LIKELIHOOD FUNCTION = -32923.4
RAW MOMENT R-SQUARE = 0.960:

ANALYSIS OF VARIANCE - FROM ZERO

	SS	DF	MS	F
REGRESSION	0.79760E+12	23.	0.34678E+11	3624.268
ERROR	0.33107E+11	3460.	0.95684E+07	
TOTAL	0.83071E+12	3483.	0.23850E+09	

TABLE A12: TOTAL NONSHELTER EXPENDITURE EQUATION FOR OWNERS (DEPENDENT VARIABLE Le_{i0})

VARIABLE NAME	ESTIMATED COEFFICIENT	STANDARD ERROR	T-RATIO 3460 DF	PARTIAL CORR.	STANDARDIZED COEFFICIENT	ELASTICITY AT MEANS
AE1	0.79686	0.12638E-01	63.053	0.7312	0.86443	0.19955
AE2	0.79569	0.81278E-02	97.897	0.8572	1.1368	0.30952
AE3	0.75070	0.81392E-02	92.232	0.8431	1.0770	0.25063
AE4	0.75682	0.94382E-02	80.186	0.8063	0.88434	0.17622
AE5	0.72497	0.17464E-01	41.512	0.5766	0.47200	0.68347E-01
D1A1	-96.123	128.80	-0.74632	-0.0127	-0.53869E-02	-0.95790E-03
D1A2	-604.21	131.02	-4.6115	-0.0782	-0.25975E-01	-0.36956E-02
D1A3	-56.271	435.27	-0.12928	-0.0022	-0.67683E-03	-0.31621E-04
D1A4	487.11	877.69	0.55499	0.0094	0.27331E-02	0.33382E-04
D2A1	471.40	166.43	2.8325	0.0481	0.15900E-01	0.17537E-02
D2A2	-271.84	108.46	-2.5064	-0.0426	-0.15930E-01	-0.31708E-02
D2A3	-350.21	191.39	-1.8298	-0.0311	-0.10172E-01	-0.10368E-02
D2A4	500.91	702.05	0.71349	0.0121	0.35871E-02	0.12358E-03
D3A1	505.11	411.03	1.2289	0.0209	0.62609E-02	0.25029E-03
D3A2	168.35	126.60	1.3298	0.0226	0.81684E-02	0.12847E-02
D3A3	33.403	146.12	0.22860	0.0039	0.13807E-02	0.20378E-03
D3A4	146.95	266.23	0.55197	0.0094	0.28983E-02	0.21148E-03
D3A5	-877.88	1173.7	-0.74796	-0.0127	-0.37104E-02	-0.78209E-04
D4A1	-328.99	187.24	-1.7571	-0.0299	-0.26826E-01	-0.66418E-02
D4A2	-34.429	117.58	-0.29280	-0.0050	-0.34545E-02	-0.99534E-03
D4A3	474.66	89.400	5.3094	0.0899	0.60436E-01	0.14460E-01
D4A4	140.35	103.64	1.3542	0.0230	0.14844E-01	0.31355E-02
D4A5	-139.77	162.54	-0.85990	-0.0146	-0.96745E-02	-0.15250E-02

R-SQUARE = 0.9165 R-SQUARE ADJUSTED = 0.9160
VARIANCE OF THE ESTIMATE = 0.84237E+07
STANDARD ERROR OF THE ESTIMATE = 2902.4
LOG OF THE LIKELIHOOD FUNCTION = -32701.6
RAW MOMENT R-SQUARE = 0.9845

ANALYSIS OF VARIANCE - FROM ZERO

	SS	DF	MS	F
REGRESSION	0.18484E+13	23.	0.80365E+11	9540.316
ERROR	0.29146E+11	3460.	0.84237E+07	
TOTAL	0.18775E+13	3483.	0.53906E+09	

TABLE A13: TENURE CHOICE EQUATION (DEPENDENT VARIABLE T)

VARIABLE NAME	ESTIMATED COEFFICIENT	ASYMPTOTIC STANDARD ERROR	T-RATIO	ELASTICITY AT MEANS	WEIGHTED AGGREGATE ELASTICITY
AVMY1	0.11221E-03	0.62818E-05	17.863	0.10231E-02	0.45486E-01
AVMY2	0.92577E-04	0.59914E-05	15.452	0.17248E-02	0.31494E-01
AVMY3	0.72519E-04	0.59142E-05	12.262	0.13868E-02	0.20933E-01
AVMY4	0.57511E-04	0.45423E-05	12.661	0.10225E-02	0.24540E-01
AVMY5	0.40870E-04	0.39077E-05	10.459	0.45143E-03	0.22573E-01
AYY1	0.92409E-06	0.51347E-05	0.17997	0.10454E-04	0.13676E-02
AYY2	-0.31100E-05	0.54656E-05	-0.56901	-0.40038E-04	-0.30417E-02
AYY3	-0.98371E-05	0.62656E-05	-1.5700	-0.10240E-03	-0.63626E-02
AYY4	-0.91811E-05	0.65293E-05	-1.4061	-0.64080E-04	-0.43176E-02
AYY5	-0.21808E-04	0.85000E-05	-2.5657	-0.50815E-04	-0.67940E-02
AGEB1	-2.5412	0.24301	-10.457	-0.84783E-03	-0.12483
AGEB2	-2.3429	0.26546	-8.8257	-0.77709E-03	-0.71613E-01
AGEB3	-1.9197	0.29366	-6.5371	-0.47969E-03	-0.36271E-01
AGEB4	-1.8794	0.27089	-6.9379	-0.38907E-03	-0.32062E-01
AGEB5	-1.3807	0.25681	-5.3764	-0.17328E-03	-0.25174E-01
D1	0.26261	0.75119E-01	3.4959	0.11735E-03	0.15960E-01
D2	0.14959	0.70295E-01	2.1280	0.67464E-04	0.65008E-02
D3	0.64101E-02	0.81716E-01	0.78443E-01	0.24052E-05	0.20545E-03
D4	0.35781	0.79118E-01	4.5224	0.10325E-02	0.10532
C	0.46669	0.13122	3.5567	0.32331E-03	0.35490E-01

MADDALA R-SQUARE 0.5088
CRAGG-UHLER R-SQUARE 0.78320
MCFADDEN R-SQUARE 0.67783
ADJUSTED FOR DEGREES OF FREEDOM 0.67645
APPROXIMATELY F-DISTRIBUTED 2.2147 WITH 19 AND 20 D.F.
CHOW R-SQUARE 0.87072

PREDICTION SUCCESS TABLE

		ACTUAL 0	ACTUAL 1
PREDICTED	0	897	117
	1	74	3366

NUMBER OF RIGHT PREDICTIONS = 0.426E+04
PERCENTAGE OF RIGHT PREDICTIONS = 0.95712

EXPECTED OBSERVATIONS AT 0 = 1001.2 OBSERVED = 971
EXPECTED OBSERVATIONS AT 1 = 3452.8 OBSERVED = 3483
SUM OF SQUARED "RESIDUALS" = 175.43
WEIGHTED SUM OF SQUARED "RESIDUALS" = 0.24991E+11